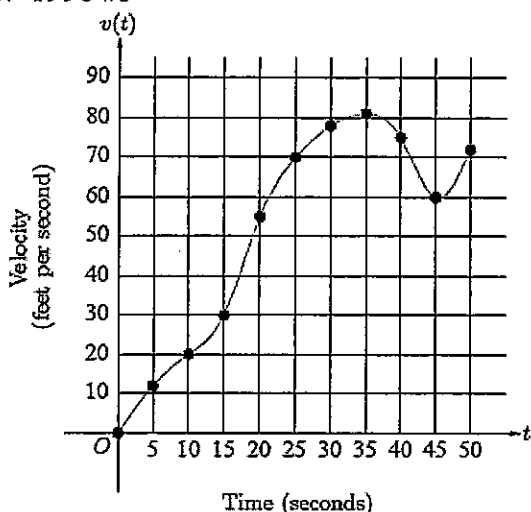


HW1: 1998 #3



$t$ (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity  $v(t)$ , in ft/sec, of a car traveling on a straight road, for  $0 \leq t \leq 50$ , is shown above. A table of values for  $v(t)$ , at 5 second intervals of time  $t$ , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
  - Find the average acceleration of the car, in  $\text{ft}/\text{sec}^2$ , over the interval  $0 \leq t \leq 50$ .
  - Find one approximation for the acceleration of the car, in  $\text{ft}/\text{sec}^2$ , at  $t = 40$ . Show the computations you used to arrive at your answer.
  - Approximate  $\int_0^{50} v(t) dt$  with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a)  $a(t) > 0$  on  $(0, 35) \cup (45, 50)$  b/c  $v'(t) > 0$  on this interval  
(or) b/c  $v(t)$  is increasing on this interval.

(b)  $\text{ave rate } \Delta \text{ in } v(t) = \frac{v(50) - v(0)}{50 - 0} = 1.44 \text{ ft}/\text{sec}^2$

(c)  $a(40) \approx \frac{v(45) - v(35)}{45 - 35} = \frac{60 - 81}{10} = -2.1 \text{ ft}/\text{sec}^2$  (Average on a small neighborhood method)

(d)  $\int_0^{50} v(t) dt \approx \text{MRAM} = \Delta t [v(5) + v(15) + v(25) + v(35) + v(45)]$   
 $= 10 [12 + 30 + 70 + 81 + 60]$   
 $= 2530 \text{ ft}$

This is the total distance traveled by the car in the first 50 sec. NOTE  $v(t) \geq 0$  on  $0 \leq t \leq 50$  (otherwise the integral is displacement)

HW2: 2004FB #3

$t$ (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.
- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.
- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?

[SEE ATTACHED SHEET]  
(on Back)

2004FB #3

a)  $\int_0^{40} v(t) dt \approx MRAM = \Delta t [9.2 + 7.0 + 2.4 + 4.3] = 10 [9.2 + 7.0 + 2.4 + 4.3] = 229$

The integral gives the total distance, in miles, that the plane flew over this 40 min interval  $[0, 40]$  b/c  $v(t) > 0$ .

b) t	0	5	10	15	20	25	30	35	40	
v(t)	7	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3	
Ave v'(t)		0.44	0.06	-0.5	-0.5	-0.42	0	0.38	0.6	← ave a(t)
			•	•						
				a(t) = v'(t)						

- On the interval  $5 < t < 15$   $a(t)$  equal 0.06 and -0.5 at least once by the M.V.T. This also means that  $a(t)$  must pass through zero at least one time on this interval (By the Intermediate Value Theorem).
- The average  $a(t)$  on the interval  $25 < t < 30$  is zero. By the M.V.T,  $a(t)$  must equal zero at least one time on this interval.

∴  $a(t) = 0$  at least two times on  $0 < t < 40$ .

\* Essentially we are looking that average  $a(t)$  changes sign. \*

c)  $f'(23) = -0.408$  miles/min<sup>2</sup>

There are many ways you can get this answer. Here is what I did

□  $y_1(x) = 6 + \cos(x/10) + 3 \sin(7x/10)$

□ Home:  $d(y_1(x), x) | x=23$

↑ type in equals button  
 ↓ button below the equals button

This sure beats any copy and paste 😊

d) Av. velocity value =  $\frac{1}{40} \int_0^{40} f(t) dt = 5.916$  miles per minute.

HW3: 2008 #2

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
  - Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
  - For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
  - The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?

a) 
$$L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8 \text{ people per hour}$$

This is "average on a small neighborhood method."

b) 
$$\frac{\int_0^4 L(t) dt}{4 - 0} = \frac{1}{4} \int_0^4 L(t) dt \approx \frac{1}{4} \text{ TRAM} = \frac{1}{4} \left[ \frac{120 + 156}{2} (1) + \frac{156 + 176}{2} (2) + \frac{176 + 126}{2} (1) \right]$$

$= 155.25 \text{ people}$

c) 
$$\begin{array}{ccccccc} 0 & 1 & 3 & 4 & 7 & 8 & 9 \\ 120 & 156 & 176 & 126 & 150 & 80 & 0 \\ \text{Av } L'(t) & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow \\ & 36 & 10 & -50 & 8 & -70 & -80 \end{array}$$

□  $\text{Ave } L'(t) = 36, 10, -50, 8, -70, -80$  at least one time on  $0 < t < 9$  by the M.V.T.

□ The average  $L'(t)$  changes sign three times and so  $L'(t)$  must equal zero at least three times by the Intermediate Value Theorem

d) 
$$\int_0^3 r(t) dt \approx 973 \text{ tickets}$$