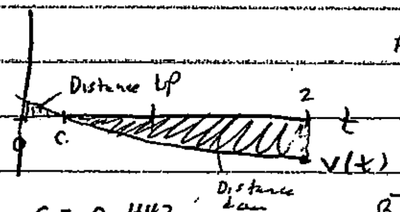
LESSON 2A HW Solutions BC: Q301 AB: Q303

HW #1] $v(t) = 1 - \tan^{-1}(e^t)$ $\quad y(0) = -1$

$v(t) = 0$ at $t = 0.443$

GRAPH



$c = 0.443$

A] Total Distance $= \int_0^c v(t)\,dt - \int_c^2 v(t)\,dt = 0.454$ m

B] displacement $= \int_0^2 v(t)\,dt = -0.361$

Position $y(2) = y(0) + \int_0^2 v(t)\,dt$

$= -1 + \int_0^2 v(t)\,dt = -1.361$ m

C] Average velocity $= \dfrac{\int_0^2 v(t)\,dt}{2-0} = -0.180$ m/s

D] Average acceleration $= \dfrac{v(2) - v(0)}{2-0} = -0.325$ m/s^2

#2 AP 2011] $v(t) = 2\sin(e^{t/4}) + 1$ $\quad v(t) = 0$ at $t = 5.196$

$a(t) = \frac{1}{2} e^{t/4} \cos(e^{t/4})$ $\quad x(0) = 2$

a) $a(5.5) = -1.359$ and $v(5.5) = -0.453$

The particle is speeding up because $a(5.5)$ is the same sign as $v(5.5)$

b) average velocity $= \dfrac{\int_0^6 v(t)\,dt}{6-0} = 1.949$

c) Total distance $= \int_0^c v(t)\,dt - \int_c^6 v(t)\,dt = 12.573$

$c = 5.196$

D) PARTICLE CHANGES DIRECTION AT $t = 5.196$

Position $x(5.196) = x(0) + \int_0^{5.196} v(t)\,dt = 2 + \int_0^{5.196} v(t)\,dt = 14.135$

#3 2009 AP]

a) $a(7.5)$ = slope of velocity at $t = 7.5 = -0.1$

b) $\int_0^{12} |v(t)|\,dt$ represents Caren's total distance traveled.

$\int_0^{12} |v(t)|\,dt = \frac{1}{2}(2)(0.2) + \frac{1}{2}(2)(0.2) + \frac{1}{2}(1)(0.3) + 1(0.3)$

$+ \frac{1}{2}(0.3+0.2)(1) + 3(0.2) + \frac{1}{2}(1)(0.2) = \underline{1.8}$ miles

c) Caren turns around at $t = 2$. At this time she changes from moving in positive direction to moving in negative direction. i.e her velocity goes from positive to negative at $t = 2$. Also $v(2) = 0$

d)

Caren's net distance traveled $= \int_0^{12} v(t)\,dt = 1.4$ miles

Larry's net distance traveld $= \int_0^{12} w(t)\,dt = 1.6$ miles

so Caren lives closer to school.

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LESSON 2B HW Solutions

4. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by
$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$.
A pumping station adds sand to the beach at a rate modeled by the function S, given by
$S(t) = \frac{15t}{1+3t}$.

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measures in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

A. How much of the sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
B. Write a function, involving an expression, for $Y(t)$, the total number of cubic yards of sand on the beach at time t.
C. Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
D. Find the total number of cubic yards of sand on the bank at time $t = 4$.

$Y(t)$ = the total number of cubic yards of sand at time t
$R(t)$ = rate out "removal"
$S(t)$ = rate in "added"

A] $\int_0^6 R(t)\,dt = 31.816$ cubic Yards

dummy variable

B] $Y(t) = Y(0) + \int_0^t [S(u) - R(u)]\,du = 2500 + \int_0^t [S(u) - R(u)]\,du$

C] $S(4) - R(4) = -1.909$ cubic yards/hr

D] $Y(4) = 2500 + \int_0^4 [S(t) - R(t)]\,dt = 2493.54$ cubic Yards

no longer need for dummy variable

LESSON 2B HW

5. AP:2003FB#2

2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

Let $A(t)$ = amount of oil in tank time t

$A(0) = 125$ $H(t)$ = rate in

$R(t)$ = rate out

a) $\int_0^{12} H(t)\,dt = \boxed{70.571 \text{ gallons}}$

b) $H(6) - R(6) = \boxed{-2.924}$ The amount of oil is falling b/c more oil is leaving than arriving at $t = 6$.

c) $A(12) = A(0) + \int_0^{12} [H(t) - R(t)]\,dt$

$= 125 + \int_0^{12} [H(t) - R(t)]\,dt = \boxed{122.026 \text{ gallons}}$

D) where is $A(t)$ min

$A'(t) = H(t) - R(t) = 0$ at $t = 11.319$

A is locally min at $t = 11.319$ b/c $A'(t)$ changes from negative to positive at $t = 11.319$

A increases for $t > 11.319$ so $A(12)$ is not a candidate.

$A(0) = 125$

$A(11.319) = 125 + \int_0^{11.319} [H(t) - R(t)]\,dt = 120.738 < 125$

A is min at $t = 11.319$.

9

LESSON 2B HW

6. AP:2002FB#2

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

$P(0) = 50$

a) $P'(9) = -0.646$ The amount of pollutant is decreasing because the rate of change is negative.

b) $P'(t) = 0$ at $t = 30.174$

$P'(t) < 0$ on $(0, 30.174)$ and $P'(t) > 0$ on $(30.174, \infty)$

$\therefore P$ is min at $t = 30.174$

c) $P(30.174) = 50 + \int_0^{30.174} P'(t)\,dt = \underline{35.104}$

Yes, the lake is safe because there is less than 40 gallons of pollutant.

D) $L(t) = P(0) + P'(0)(t)$

$L(t) = 50 - 2(t) = 40$

$-2t = -10$

$t = 5$ hrs.

10

LESSON 2B HW

7. AP:2010#1

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

$g(t)$ = rate out (snow off)

$f(t)$ = rate in (snow on)

$A(t)$ = amount of snow on driveway (ft^3) at time t

a) $\int_0^6 f(t)\,dt = 142.275$ cubic feet

b) $f(8) - g(8) = f(8) - 108 = -59.583$ cubic feet/hr

c) $$\boxed{h(t) = \begin{cases} 0 & 0 \le t \le 6 \\ 125(t-6) & 6 < t \le 7 \\ 125 + 108(t-7) & 7 < t \le 9 \end{cases}}$$

$\left[125u\right]_6^t = 125t - 125(6) = 125(t-6)$

Details: for $t \in [0,6]$ $\quad h(t) = h(0) + \int_0^t 0\,du = 0$

for $t \in (6,7]$ $\quad h(t) = h(6) + \int_6^t 125\,du = 0 + \int_6^t 125\,du = 125(t-6)$

for $t \in (7,9]$ $\quad h(t) = h(7) + \int_7^t 108\,du = 125 + \int_7^t 108\,du = 125 + 108(t-7)$

$$h(t) = \begin{cases} 0 & 0 \le t \le 6 \\ 0 + \int_6^t 125\,du & 6 < t \le 7 \\ 125 + \int_7^t 108\,du & 7 < t \le 9 \end{cases}$$

← you may have put this ... But the question did not ask for an integral expression in your answer.

D] $A(9) = 0 + \underbrace{\int_0^9 f(t)\,dt}_{\text{amount in}} \underbrace{- 125}_{\text{snow off b/w } 6\to 7} \underbrace{- 108(2)}_{\text{Snow off } 7\to 9} = 26.335$ cubic feet

11