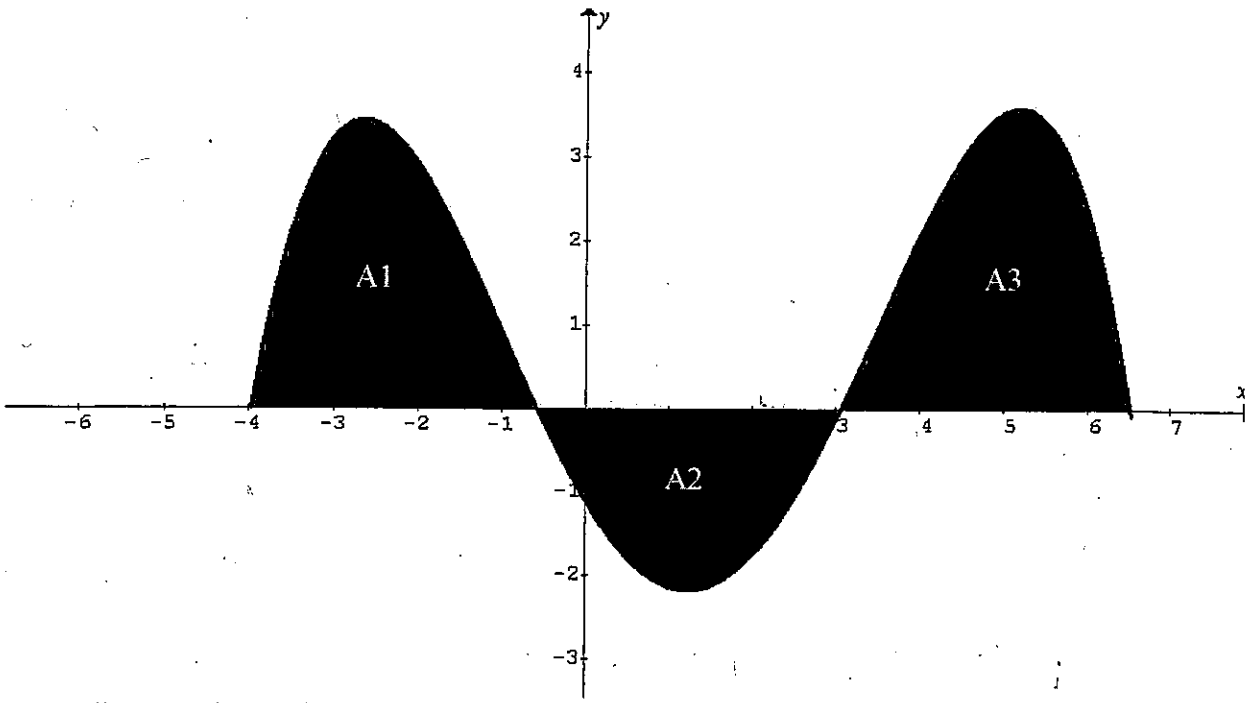
Homework Solutions

CALCULUS AP AB – Q303 CH5A: (Lesson 1-A) AREA and INTEGRAL
BC – Q301

AB: Q303 CH5A – LESSON (1A) HOMEWORK

1.



In the diagram above, the values of the areas A1, A2, and A3 bounded by the graph of $f(x)$ and the x-axis, are 7, 5, and 8 square units respectively. $f(x)$ has zeros at -4, -0.6, 3, and 6.5.

Calculate the following definite integrals:

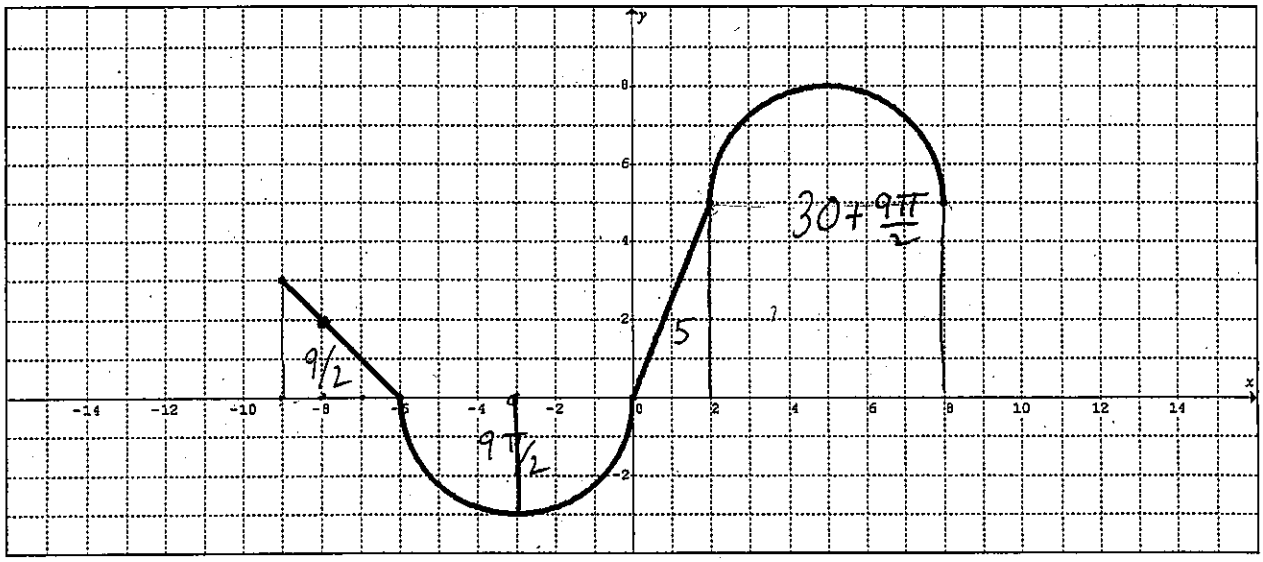
A. $\int_{-0.6}^{3} f(x)dx = -A2 = \boxed{-5}$

B. $\int_{-4}^{6.5} f(x)dx = A1 - A2 + A3 = 7 - 5 + 8 = \boxed{10}$

C. $\int_{3}^{-4} f(x)dx = -\int_{-4}^{3} f(x)\,dx = -[A1 - A2] = -[7-5] = -(2) = \boxed{-2}$

D. $\int_{-4}^{-0.6} f(x)dx - \int_{-0.6}^{3} f(x)dx + \int_{3}^{6.5} f(x)dx = A1 + A2 + A3 = 7+5+8 = \boxed{20}$

2. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

$A = \frac{1}{2}(3)(3) + \frac{1}{2}\pi(3)^2 + \frac{1}{2}(2)(5) + \left[30 + \frac{1}{2}\pi(3)^2\right]$

$= \frac{9}{2} + \frac{9\pi}{2} + \frac{10}{2} + \frac{60}{2} + \frac{9\pi}{2} = \boxed{\frac{79}{2} + 9\pi}$

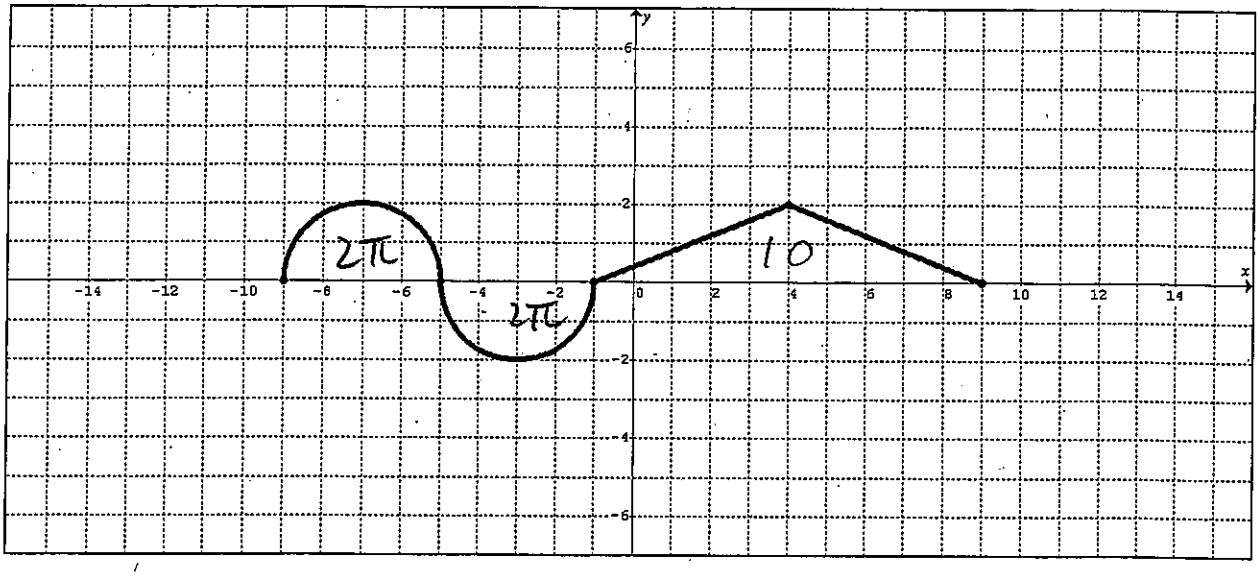
B. $\int_{-9}^{8} f(x)dx = \frac{9}{2} - \frac{9\pi}{2} + \frac{10}{2} + \frac{60}{2} + \frac{9\pi}{2} = \boxed{\frac{79}{2}}$

C. $\int_{-3}^{2} f(x)dx = \boxed{-\frac{9\pi}{4} + 5}$

D. $\int_{0}^{-9} f(x)dx = -\int_{-9}^{0} f(x)\,dx = -\left[9/2 - \frac{9\pi}{2}\right] = \boxed{\frac{9\pi}{2} - \frac{9}{2}}$

E. $7 + \int_{-8}^{-5} f(x)dx = 7 + \frac{1}{2}(2)(2) = 7 + 2 = \boxed{9}$

3. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

$A = \frac{1}{2}\pi(2)^2 + \frac{1}{2}\pi(2)^2 + \frac{1}{2}(10)(2) = \boxed{4\pi + 10}$

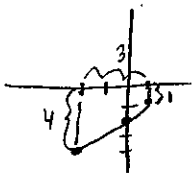
B. $\int_{-9}^{9} f(x)dx = 2\pi - 2\pi + 10 = \boxed{10}$

C. $\int_{-9}^{-1} f(x)dx = 2\pi - 2\pi = 0$

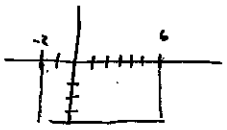
D. $\int_{-3}^{4} f(x)dx = \boxed{-\pi + 5}$

E. $10 - \int_{2}^{2} f(x)dx = 10 - 0 = \boxed{10}$

4. Evaluate the following by appealing to geometry.

A. $\int_{-2}^{1}(x-2)dx$ 

$= -A$
$= -\frac{1+4}{2} \cdot 3$
$= \boxed{\frac{-15}{2}}$

B. $\int_{-2}^{6} -4dx$ 
$= -A$
$= -(8 \cdot 4)$
$= \boxed{-32}$

5. Suppose that f and g are continuous functions and that …

$\int_{1}^{2} f(x)dx = -4$, $\int_{1}^{5} f(x)dx = 6$, and $\int_{1}^{5} g(x)dx = 8$

Evaluate each of the following integrals:

A. $\int_{2}^{2} g(x)dx = \boxed{0}$

B. $\int_{5}^{1} g(x)dx = -\int_{1}^{5} g(x)\,dx = \boxed{-8}$

C. $\int_{1}^{2} 3f(x)dx = 3\int_{1}^{2} f(x)\,dx = 3(-4) = \boxed{-12}$

D. $\int_{2}^{5} f(x)dx = \int_{2}^{1} f(x)\,dx + \int_{1}^{5} f(x)\,dx = -\int_{1}^{2} f(x)\,dx + \int_{1}^{5} f(x)\,dx = 4 + 6 = \boxed{10}$

E. $\int_{1}^{5} [f(x)-g(x)]dx = \int_{1}^{5} f(x)\,dx - \int_{1}^{5} g(x)\,dx = 6 - 8 = \boxed{-2}$

F. $\int_{1}^{5} [4f(x)-g(x)]dx = 4\int_{1}^{5} f(x)\,dx - \int_{1}^{5} g(x)\,dx = 4(6) - 8 = \boxed{16}$