BC Q203 L3 HW SOLUTIONS.

1. Let $\frac{dP}{dt} = 0.006P(200 - P)$ with initial condition $P = 8$ people when $t = 0$ years.

A. Write the function $P$(t).

B. What is the size of the population when it is growing its fastest?

C. What is the rate at which the population is growing when it is growing the fastest?

D. Find $\lim_{t\to\infty} P(t)$

A] $P = \frac{200}{1 + c^* e^{-1.2t}}$

$8 = \frac{200}{1 + c^*}$

$1 + c^* = \frac{200}{8}$

$1 + c^* = 25$

$c^* = 24$

$$\boxed{P = \frac{200}{1 + 24e^{-1.2t}}}$$

B] $P$ is growing fastest when $P = 100$ " $\frac{200}{2}$ "

C] $P = 100$ : $\frac{dP}{dt} = 0.006(100)(200-100)$

$= 0.006(10000)$

$= 60$ people/year

D] $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{200}{1 + 24e^{-1.2t}} = 200$

2. Let $\frac{dP}{dt} = 1200P - 100P^2$ with initial condition $P = 4$ eggs when $t = 0$ months.

A. Write the function $P(t)$.

B. What is the size of the population when it is growing its fastest?

C. What is the rate at which the population is growing when it is growing the fastest?

D. Find $\lim_{t\to\infty} \boldsymbol{P(t)}$

$$\frac{dP}{dt} = 100P(12 - P) \quad K = 100 \quad L = 12$$

A] $$P = \frac{12}{1 + c^* e^{-1200t}}$$

$$4 = \frac{12}{1 + c^*}$$

$$1 + c^* = 3$$

$$c^* = 2$$

$$\boxed{P = \frac{12}{1 + 2e^{-1200t}}}$$

B] $P$ is growing fastest when $P = 6$

C] $P = 6$: $\frac{dP}{dt} = 100(6)(12 - 6) = 3600$ eggs/month

D] $$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{12}{1 + 2e^{-1200t}} = 12$$

K / L

3. A certain rumor spreads through a community at the rate $\frac{dy}{dt} = 2y(1-y)$, where $y$ is the proportion of the population that has heard the rumor at time $t$ hours.

(a) What proportion of the population has heard the rumor when it is spreading the fastest?

(b) If ten percent of the people have heard the rumor at time $t = 0$, find $y$ as a function of $t$.

(c) At what time $t$ is the rumor spreading the fastest?

(a) $y$ is growing fastest when $\boxed{y = \frac{1}{2}}$ (or 50%)

(b) $y = \frac{1}{1 + C^* e^{-2t}}$

$0.1 = \frac{1}{1 + C^*}$

$1 + C^* = \frac{1}{0.1} = 10$

$C^* = 9$

$$\boxed{y = \frac{1}{1 + 9e^{-2t}}}$$

(c) $\frac{1}{2} = \frac{1}{1 + 9e^{-2t}}$

$2 = 1 + 9e^{-2t}$

$1 = 9e^{-2t}$

$\frac{1}{9} = e^{-2t}$

$-2t = \ln\left(\frac{1}{9}\right)$

$t = \boxed{\frac{\ln\left(\frac{1}{9}\right)}{-2}}$ hours

or

$\boxed{\frac{\ln(9)}{2}}$

4. $P'(0) = P'(0,3) = 100(3) - 5(3)^2 = \boxed{255}$

$$\frac{d^2P}{dt^2} = \frac{d}{dt}\left(\frac{dP}{dt}\right) = \frac{d}{dt}\left(100P - 5P^2\right)$$

$$= 100\frac{dP}{dt} - 10P\frac{dP}{dt}$$

$$P''(0) = P''\left(x=0,\ y=3,\ \frac{dP}{dt}=255\right) = 100(255) - 10(3)(255)$$

$$= \boxed{17850}$$