BC: Q204 HW SOLUTIONS — LESSON 1

1. $\frac{dy}{dt} = 5(10-y)$

$\int \frac{dy}{10-y} = \int 5\,dt$

$-\ln|10-y| = 5t + c$

$\ln|10-y| = -5t + c$

$|10-y| = Ce^{-5t}$

$10-y = c^* e^{-5t}$

$y = 10 - C^* e^{-5t}$

$7 = 10 - C^* e^0 \quad \therefore C^* = 3$

$\boxed{y = 10 - 3e^{-5t}}$

2. $\frac{dy}{dt} = \frac{1}{2}(5-y)$

$\int \frac{dy}{5-y} = \int \frac{1}{2}\,dt$

$-\ln|5-y| = \frac{1}{2}t + c$

$\ln|5-y| = \frac{-t}{2} + c$

$|5-y| = Ce^{-t/2}$

$5-y = c^* e^{-t/2}$

$y = 5 - C^* e^{-t/2}$

$8 = 5 - C^* e^0 \quad \therefore C^* = -3$

$\boxed{y = 5 + 3e^{-t/2}}$

3. $\frac{dy}{dt} = 8(2+y)$

$\int \frac{dy}{2+y} = \int 8\,dt$

$\ln|2+y| = 8t + C$

$|2+y| = Ce^{8t}$

$2+y = C^* e^{8t}$

$y = -2 + C^* e^{8t}$

$1 = -2 + C^* e^0 \quad C^* = 3$

$\boxed{y = -2 + 3e^{8t}}$

4. $\frac{dy}{dt} = -4(3-y)$

$\int \frac{dy}{3-y} = \int -4\,dt$

$-\ln|3-y| = -4t + C$

$\ln|3-y| = 4t + c$

$|3-y| = Ce^{4t}$

$3-y = C^* e^{4t}$

$y = 3 - C^* e^{4t}$

$6 = 3 - C^* e^0 \quad C^* = -3$

$\boxed{y = 3 + 3e^{4t}}$

5. $\frac{dy}{dt} = 0.4y$

$\int \frac{dy}{y} = \int 0.4\,dt$

$\ln|y| = 0.4t + c$

$|y| = Ce^{0.4t}$

$y = c^* e^{0.4t}$

$6 = C^* e^0 \quad \therefore C^* = 6$

$\boxed{y = 6e^{0.4t}}$

(1) $\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + C$

$y = \pm\sqrt{x^2 + c}$

$2 = +\sqrt{1+c} \quad \therefore \; c = 3$

$\boxed{y = \sqrt{x^2+3}}$ $\quad x \in \mathbb{R}$

(2) $\frac{dy}{dx} = \frac{y}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = \ln|x| + C$

$|y| = C e^{\ln|x|}$

$|y| = C|x|$

$y = C^*|x|$

$2 = C^*|2| \quad \therefore \; C^* = 1$

$\boxed{y = |x|}$ $\quad x \in R$

(3) $\frac{dy}{dx} = xy$

$\int \frac{dy}{y} = \int x\,dx$

$\ln|y| = \frac{x^2}{2} + c$

$|y| = Ce^{x^2/2}$

$y = C^* e^{x^2/2}$

$2 = C^* e^{1/2} \quad \therefore \; C^* = \frac{2}{e^{1/2}}$ or $2e^{-1/2}$

$\boxed{y = 2e^{-1/2} \cdot e^{x^2/2}}$

or

$\boxed{y = 2e^{\frac{x^2-1}{2}}}$ $\quad x \in \mathbb{R}$

(4) $\frac{dy}{dx} = \frac{1}{xy}$

$\int y\,dy = \int \frac{1}{x}\,dx$

$\frac{y^2}{2} = \ln|x| + c$

$y^2 = 2\ln|x| + c$

$y = \pm\sqrt{2\ln|x| + c}$

$-4 = -\sqrt{2\ln(1) + c}$

$4 = \sqrt{c} \quad \therefore \; c = 16$

$\boxed{y = -\sqrt{2\ln|x| + 16}}$ $\quad \cancel{x \in \mathbb{R}}$ *

(5) $\frac{dy}{dx} = (y+5)(x+2)$

$\int \frac{dy}{y+5} = \int (x+2)\,dx$

$\ln|y+5| = \frac{x^2}{2} + 2x + c$

$|y+5| = Ce^{\left(\frac{x^2}{2}+2x\right)}$

$y+5 = C^* e^{\left(\frac{x^2}{2}+2x\right)}$

$y = -5 + C^* e^{\left(\frac{x^2}{2}+2x\right)}$

$1 = -5 + C^* e^0$

$\therefore C^* = 6$

$\boxed{y = -5 + 6e^{\left(\frac{x^2}{2}+2x\right)}}$

$x \in \mathbb{R}$

$2\ln|x| + 16 \geq 0$

$|x| \geq e^{-8}$

$* \; D: (-\infty, -e^{-8}] \cup [e^{-8}, \infty)$

(6) $\frac{dy}{dx} = (\cos x)e^{(y+\sin x)} = (\cos x)e^{y} \cdot e^{(\sin x)}$

$\boxed{u = \sin x}$

$\int \frac{dy}{e^y} = \int (\cos x)e^{(\sin x)} dx \longrightarrow \int e^{-y} dy = \int e^{u} du$

$-e^{-y} = e^{u} + C \longrightarrow -e^{-y} = e^{\sin x} + C$

$e^{-y} = -e^{\sin x} + C \longrightarrow -y = \ln\left(-e^{\sin x} + C\right)$

Key Fact $\boxed{\ln(1) = 0}$

$y = -\ln(-e^{\sin x} + c) \rightarrow 0 = -\ln(-e^{0} + c)$

$0 = -\ln(-1 + C) \quad \therefore C = 2$

$y = -\ln(-e^{\sin x} + 2)$ or $y = -\ln(2 - e^{\sin x})$

Domain: $\{2 - e^{\sin x} > 0\} \equiv \{e^{\sin x} < 2\} \equiv \{\sin x < \ln(2)\}$

(7) $\frac{dy}{dx} = -2xy^2$

$\int \frac{dy}{y^2} = \int -2x \, dx$

$\int y^{-2} dy = \int -2x \, dx$

$-y^{-1} = -x^2 + c$

$y^{-1} = x^2 + c$

$\frac{1}{y} = x^2 + C$

$y = \frac{1}{x^2 + c}$

$\frac{1}{4} = \frac{1}{1 + c} \quad \therefore c = 3$

$\boxed{y = \frac{1}{x^2 + 3}} \quad x \in \mathbb{R}$

1. WOLVES $\quad \frac{dP}{dt} = k(800 - P)$

A] $\int \frac{dP}{800-P} = \int k\,dt$

$-\ln|800-P| = kt + C$

$\ln|800-P| = -kt + C$

$|800-P| = Ce^{-kt}$

$800 - P = C^* e^{-kt}$

$P = 800 - C^* e^{-kt}$

$500 = 800 - C^* e^{\cancel{0}}$

$\therefore C^* = 300$

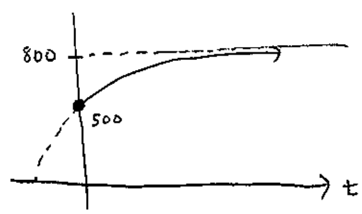
$\boxed{P = 800 - 300e^{-kt}}$

B] $700 = 800 - 300e^{-2k}$

$\frac{1}{3} = e^{-2k}$

$k = \frac{\ln(\frac{1}{3})}{-2}$

$\boxed{P = 800 - 300\left(\frac{1}{3}\right)^{\frac{t}{2}}}$



C] $P(6) = 800 - 300\left(\frac{1}{3}\right)^3$

$= 800 - \frac{300}{27}$

$= 800 - \frac{100}{9}$ approx 11

≈ 789 wolves

D] $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} 800 - \cancel{300\left(\frac{1}{3}\right)^t} \; 0$

$= 800$ wolves

2. MEL SNAKE

Let M = Length (inches) of a Mel snake at time t months

$t = 0 \rightarrow M = 2$

$t = 4 \rightarrow M = 21$

$\frac{dM}{dt} = k(40 - M)$

A] $\int \frac{dM}{40-M} = \int k\,dt$

$-\ln|40-M| = kt + c$

$\ln|40-M| = -kt + c$

$|40-M| = ce^{-kt}$

$40 - M = c^* e^{-kt}$

$M = 40 - c^* e^{-kt}$

$2 = 40 - c^* e^0 \quad \therefore c^* = 38$

$\boxed{M = 40 - 38e^{-kt}}$

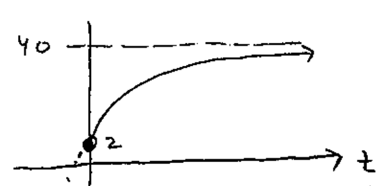
B] $21 = 40 - 38e^{-4k}$

$\frac{19}{38} = e^{-4k}$

$\frac{1}{2} = e^{-4k}$

$k = \frac{\ln(\frac{1}{2})}{-4}$

$\boxed{M = 40 - 38\left(\frac{1}{2}\right)^{t/4}}$



C] $\frac{141}{4} = 40 - 38\left(\frac{1}{2}\right)^{t/4}$

$\left(\frac{141}{4} - \frac{160}{4}\right) = -38\left(\frac{1}{2}\right)^{t/4}$

$-\frac{19}{4} = -38\left(\frac{1}{2}\right)^{t/4}$

$\frac{19}{4(38)} = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\frac{t}{4} = 3$

$\boxed{t = 12 \text{ months}}$

3. T = temp. (°C) of water at time t minutes

$$\frac{dT}{dt} = k(18 - T) \qquad \begin{array}{l} t=0 \longrightarrow T = 98 \\ t=5 \longrightarrow T = 38 \end{array}$$

A] $\displaystyle\int \frac{dT}{18-T} = \int k\,dt$

$-\ln|18-T| = kt + C$

$\ln|18-T| = -kt + C$

$|18-T| = Ce^{-kt}$

$18 - T = C^* e^{-kt}$

$T = 18 - C^* e^{-kt}$

$98 = 18 - C^* e^0 \quad \therefore C^* = -80$

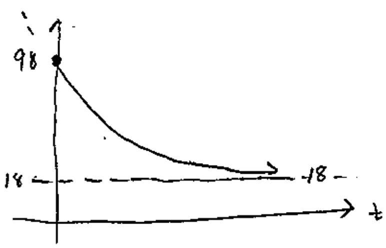
$\boxed{T = 18 + 80e^{-kt}}$

B] $38 = 18 + 80e^{-5k}$

$\frac{20}{80} = e^{-5k}$

$k = \frac{\ln(\frac{1}{4})}{-5}$

$\boxed{T = 18 + 80\left(\frac{1}{4}\right)^{t/5}}$



C] $20 = 18 + 80\left(\frac{1}{4}\right)^{t/5}$

$\left(\frac{2}{80}\right) = \left(\frac{1}{4}\right)^{t/5}$

$\frac{1}{40} = \left(\frac{1}{4}\right)^{t/5}$

$\ln\left(\frac{1}{40}\right) = \frac{t}{5}\ln\left(\frac{1}{4}\right)$

$\frac{t}{5} = \frac{\ln(\frac{1}{40})}{\ln(\frac{1}{4})}$

$\boxed{t = \frac{5\ln(\frac{1}{40})}{\ln(\frac{1}{4})} \text{ min}}$

4. $t = 0 \quad V = 100$

$\displaystyle\int \frac{dG}{200 - G} = \int -\ln\left(\frac{1}{10}\right) dt$

$-\ln|200 - G| = -\ln\left(\frac{1}{10}\right)t + c$

$\ln|200 - G| = \ln\left(\frac{1}{10}\right) + c$

$|200 - G| = ce^{\ln(\frac{1}{10})}$

$200 - G = c^*\left(\frac{1}{10}\right)^t$

$G = 200 - c^*\left(\frac{1}{10}\right)^t$

$100 = 200 - c^* \quad \therefore c^* = 100$

$\boxed{G = 200 - 100\left(\frac{1}{10}\right)^t}$

A] $G(2) = 200 - 100\left(\frac{1}{10}\right)^2 = \boxed{199 \text{ cm}^3}$

B] $\lim\limits_{t\to\infty} G(t) = \lim\limits_{t\to\infty} 200 - 100\cancel{\left(\frac{1}{10}\right)^t}^{\,0} = \boxed{200 \text{ cm}^3}$