INTEGRATION BY PARTS — SUPPLEMENTAL HW SOLUTIONS

① $\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + c$

$u = x \quad dv = e^{-x}dx$
$du = dx \quad v = -e^{-x}$

② $\int x\sec x\tan x\,dx = x\sec x - \int \sec x\,dx = x\sec x - \ln|\sec x + \tan x| + c$

like in class u-sub.

$u = x \quad dv = \sec x\tan x\,dx$
$du = dx \quad v = \sec x$

③ $\int \cot^{-1}x\,dx = x\cot^{-1}x + \int \frac{x}{1+x^2}dx = x\cot^{-1}x + \frac{1}{2}\ln|1+x^2| + c$

u-substitution

$u = \cot^{-1}x \quad dv = dx$
$du = \frac{-1\,dx}{1+x^2} \quad v = x$

$u = 1+x^2$
$du = 2x\,dx$

④ $\int e^{-x}\sin x\,dx = -e^{-x}\cos x - \int e^{-x}\cos x\,dx = -e^{-x}\cos x - \left[e^{-x}\sin x + \int e^{-x}\sin x\,dx\right]$

$u = e^{-x} \quad dv = \sin x\,dx$
$du = -e^{-x}dx \quad v = -\cos x$

$u = e^{-x} \quad dv = \cos x$
$du = -e^{-x}dx \quad v = \sin x$

$= -e^{-x}\cos x - e^{-x}\sin x - \int e^{-x}\sin x\,dx$

$\therefore \quad 2\int e^{-x}\sin x\,dx = -e^{-x}\cos x - e^{-x}\sin x + c$

$\int e^{-x}\sin x\,dx = \frac{-e^{-x}}{2}\cos x - \frac{e^{-x}}{2}\sin x + c$

(5) $\int \sin x \ln(\cos x)\,dx = -\cos x \ln(\cos x) - \int \sin x\,dx$

$u = \ln(\cos x) \quad dv = \sin x\,dx$ $\qquad = -\cos x \ln(\cos x) + \cos x + C$

$du = \frac{1}{\cos x} \cdot (-\sin x)\,dx \quad v = -\cos x$

(6) $\int \csc^3 x\,dx = \int \csc x \cdot \csc^2 x\,dx = -\csc x \cot x - \int \csc x \cot^2 x\,dx$

$u = \csc x \quad dv = \csc^2 x\,dx$ $\qquad = -\csc x \cot x - \int \csc x(\csc^2 x - 1)\,dx$

$du = -\csc x \cot x\,dx \quad v = -\cot x$ $\qquad = -\csc x \cot x - \int \csc^3 x + \int \csc x\,dx$

$2\int \csc^3 x\,dx = -\csc x \cot x + \int \csc x\,dx$

$\frac{\csc x(\csc x + \cot x)}{\csc x + \cot x}$ $\qquad 2\int \csc^3 x\,dx = -\csc x \cot x - \ln|\csc x + \cot x| +$

$\int \csc^3 x\,dx = -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| +$

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(7) $\int \cos\sqrt{x}\,dx = \int 2\sqrt{x} \cos w\,dw = \int 2w \cos w\,dw$

$w = \sqrt{x}$ $\qquad = 2\int w \cos w\,dw$

$dw = \frac{dx}{2\sqrt{x}}$

$u = w \quad dv = \cos w$

$du = dw \quad v = \sin w$

$= 2\left[w \sin w - \int \sin w\,dw\right]$

$= 2w \sin w + 2\cos w + C$

$= 2\sqrt{x} \sin\sqrt{x} + 2\cos\sqrt{x} + C$

ALT:

$w = \sqrt{x}$ $\qquad \int \cos\sqrt{x}\,dx = \int \cos w\,(2w)\,dw$

$w^2 = x$ $\qquad = 2\int w \cos w\,dw$

$2w\,dw = dx$

now use By Parts!

(7)

$$\int e^{4x} \sin 5x \, dx = -\frac{1}{5} e^{4x} \cos(5x) + \int \frac{4e^{4x}}{5} \cos(5x)\, dx$$

$$\boxed{\begin{array}{ll} u = e^{4x} & dv = \sin 5x \, dx \\ du = 4e^{4x} dx & v = \frac{-\cos(5x)}{5} \end{array}}$$

$$= -\frac{1}{5} e^{4x} \cos(5x) + \frac{4}{5} \int e^{4x} \cos(5x)\, dx$$

$$\begin{array}{ll} u = e^{4x} & dv = \cos(5x) dx \\ du = 4e^{4x} dx & v = \frac{1}{5} \sin(5x) \end{array}$$

$$= -\frac{1}{5} e^{4x} \cos(5x) + \frac{4}{5} \left[\frac{1}{5} e^{4x} \sin(5x) - \frac{4}{5} \int e^{4} \sin(5x)\, dx \right]$$

$$= -\frac{1}{5} e^{4x} \cos(5x) + \frac{4}{25} e^{4x} \sin(5x) - \frac{16}{25} \int e^{4} \sin(5x)\, dx$$

add to both sides

$$\frac{41}{25} \int e^{4x} \sin 5x \, dx = -\frac{1}{5} e^{4x} \cos(5x) + \frac{4}{25} e^{4x} \sin(5x) + C$$

$$\boxed{\int e^{4x} \sin 5x \, dx = -\frac{5}{41} e^{4x} \cos(5x) + \frac{4}{41} e^{4x} \sin(5x) + C}$$

25 ÷ 4

2+

$$\frac{4}{25} \cdot \frac{25}{41}$$