u-Substitution HW Solutions

1. $$\int x^2 \sqrt[3]{3x^3+7}\,dx = \frac{1}{9}\int u^{1/3}\,du = \frac{1}{9}\cdot\frac{3}{4}u^{4/3}+c = \frac{1}{12}u^{4/3}+c = \boxed{\frac{1}{12}(3x^3+7)^{4/3}+c}$$

$u = 3x^3+7$

$du = 9x^2\,dx$

$dx = du/(9x^2)$

2. $$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = 2\int u^3\,du = \frac{2}{4}u^4+c = \boxed{\frac{1}{2}(1+\sqrt{x})^4+c}$$

$u = (1+\sqrt{x})$

$du = \frac{1}{2}(x)^{-1/2}\,dx$

$dx = 2\sqrt{x}\,du$

3. $$\int n^2\sqrt{n^3-1}\,dn = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2}+c = \boxed{\frac{2}{9}(n^3-1)^{3/2}+c}$$

$u = n^3-1$

$du = 3n^2\,dn$

$dn = du/3n^2$

4. $$\int \cos(3x)\sqrt[3]{\sin(3x)}\,dx = \frac{1}{3}\int u^{1/3}\,du = \frac{1}{3}\cdot\frac{3}{4}u^{4/3}+c = \boxed{\frac{1}{4}(\sin(3x))^{4/3}+c}$$

$u = \sin 3x$

$du = 3\cos 3x\,dx$

$dx = du/(3\cos 3x)$

5. $$\int \frac{\cos t}{(1-\sin t)^2}\,dt = -\int u^{-2}\,du = -\frac{u^{-1}}{-1}+c = (1-\sin t)^{-1}+c = \boxed{\frac{1}{(1-\sin t)}+c}$$

$u = 1-\sin t$

$du = -\cos t\,dt$

$dt = du/(-\cos t)$

6. $$\int \sec^2(3x)\tan(3x)\,dx = \int \sec(3x)\sec(3x)\tan(3x)\,dx = \frac{1}{3}\int u\,du$$

$u = \sec 3x$

$du = \sec 3x \tan 3x \cdot 3\,dx$

$dx = \frac{du}{3(\sec 3x \tan 3x)}$

$$= \frac{1}{3}\,\frac{u^2}{2}+c = \frac{u^2}{6}+c = \boxed{\frac{\sec^2(3x)}{6}+c}$$

OR $u = \tan 3x$

$du = 3\sec^2(3x)\,dx$

$\frac{1}{3}\int u\,du = \frac{1}{6}u^2+c$

$= \frac{1}{6}(\tan^2(3x))+c$

7. $$\int x\cot(x^2)\csc(x^2)\,dx = \frac{1}{2}\int \cot u \csc u\,du = -\frac{1}{2}\csc u + c = \boxed{-\frac{1}{2}\csc(x^2)+c}$$

$u = x^2$

$du = 2x\,dx$

$dx = du/2x$

OR $\int x\,\frac{\cos(x^2)}{\sin(x^2)}\cdot\frac{1}{\sin(x^2)}\,dx$

$= \int x\,\frac{\cos(x^2)}{\sin^2(x^2)}\,dx$

$u = \sin(x^2)$

$du = 2x\cos(x^2)\,dx$

$= \frac{1}{2}\int \frac{1}{u^2}\,du = \frac{1}{2}\,\frac{u^{-1}}{-1}+c = -\frac{1}{2}\,\frac{1}{u}+c = -\frac{1}{2}\csc(x^2)+c$

8. $f(x) = \int f'(x)\,dx = \int \sqrt[3]{3x+2}\,dx = \int (3x+2)^{1/3}\,dx$

$u = 3x+2$

$du = 3\,dx$

$= \frac{1}{3}\int u^{1/3}\,du = \frac{1}{3}\frac{3}{4}u^{4/3} + c$

$= \frac{1}{4}(3x+2)^{4/3} + c$

$f(2) = \frac{(8)^{4/3}}{4} + c = 9 \rightarrow 4 + c = 9 \therefore c = 5$

$\boxed{f(x) = \frac{1}{4}(3x+2)^{4/3} + 5}$

9. $\frac{dy}{dx} = x\sqrt{x^2+5} \rightarrow dy = x\sqrt{x^2+5}\,dx$

$u = x^2+5$

$\int dy = \int x\sqrt{x^2+5}\,dx$

$y = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{2}{3}u^{3/2} + c = \frac{1}{3}(x^2+5)^{3/2} + c$

$y(2) = \frac{1}{3}(9)^{3/2} + c = 9 + c = 12 \quad \therefore c = 3$

$\boxed{y = \frac{1}{3}(x^2+5)^{3/2} + 3}$

10. $\int \frac{1}{2x+7}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|2x+7| + c$

$u = 2x+7$

$du = 2\,dx$

11. $\int \frac{4x}{x^2-9} = 2\int \frac{1}{u}\,du = 2\ln|x^2-9| + c$

$u = x^2-9$

$du = 2x\,dx$

12. $\int e^{-4x}\,dx = -\frac{1}{4}e^{-4x} + c$

13. $\int \tan(2x)\,dx = \int \frac{\sin(2x)}{\cos(2x)}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du$

$u = \cos(2x)$

$du = -2\sin(2x)$

$= -\frac{1}{2}\ln|\cos 2x| + c$

14. $\int \frac{x-2}{x^2-4x+9}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|x^2-4x+9| + c$

$u = x^2-4x+9$

$du = (2x-4)\,dx$

15. $\int \frac{\ln x}{x}\,dx = \int u\,du$

$u = \ln x$

$du = \frac{dx}{x}$

$= \frac{u^2}{2} + c$

$= \frac{\ln^2 x}{2} + c$

#16

factor out

$7\int \frac{dx}{(6x)\sqrt{(6x)^2-1}}$ $\quad u = 6x$

$= \frac{7}{6}\sec^{-1}(6x) + C$

"BOOK OF MEMORIES"