1. A f is cont on $[0,1]$ ✓

f is diff on $(0,1)$ ✓

M.V.T APPLIES

$$\text{Ave rate}\Delta_{[0,1]} = \frac{f(1)-f(0)}{1-0} = \frac{2-(-1)}{1} = 3$$

$$\text{ins rate}\Delta = f'(x) = 2x+2$$

$$2x+2 = 3$$

$$\boxed{x = \tfrac{1}{2}} \in (0,1) \checkmark$$

B f is cont on $[2,4]$ ✓

f is diff on $(2,4)$ ✓

M.V.T APPLIES

$$\text{A.rate}\Delta_{[2,4]} = \frac{f(4)-f(2)}{4-2} = \frac{\ln(3)-\ln(1)}{2}$$

$$= \frac{\ln 3}{2}$$

$$\text{I. rate}\Delta = f'(x) = \frac{1}{x-1}$$

$$\frac{1}{x-1} = \frac{\ln(3)}{2} \rightarrow x-1 = \frac{2}{\ln 3}$$

$$\boxed{x = \frac{2}{\ln 3} + 1} \in (2,4) \checkmark$$

C.

$$\text{Ave rate}\Delta_{[1.5,3]} = \frac{f(3)-f(1.5)}{3-1.5} = -1.194$$

$$\text{ins rate}\Delta = f'(x)$$

$f'(x) = -1.194$ at $\begin{cases} x = 1.764 \in (1.5,3) \\ x = 2.540 \in (1.5,3) \end{cases}$

* NOTES *

OPTION 1

$y2 = [\text{paste } f'(x)]$

$y3 = -1.194$

Find intersections

OPTION 2

$y2 = [\text{paste } f'(x)] + 1.194$

Find zeros

1989 AB-1 $f(x) = x^3 - 7x + 6$

a) $f(x) = 0 \quad x^3 - 7x + 6 = 0$

$x = -3 \qquad x = 1$

$-3 \mid 1 \;\; 0 \;\; -7 \;\; 6$ ← synthetic Division → $1 \mid 1 \;\; 0 \;\; -7 \;\; 6$

$\downarrow \; -3 \;\; 9 \;\; -6$ OR $\downarrow \; 1 \;\; 1 \;\; -6$

$1 \;\; -3 \;\; 2 \;\; \boxed{0}$ $\qquad 1 \;\; 1 \;\; -6 \;\; \boxed{0}$

$(x+3)(x^2 - 3x + 2) = 0$ OR $(x-1)(x^2 + x - 6) = 0$

$(x+3)(x-2)(x-1) = 0$ $(x-1)(x+3)(x-2) = 0$

$\boxed{x = -3, 2, 1}$ $\boxed{x = -3, 2, 1}$

b) $f'(x) = 3x^2 - 7$

$f'(-1) = 3 - 7 = -4$

$f(-1) = -1 + 7 + 6 = 12$

$\boxed{y - 12 = -4(x + 1)}$

c) $f_{av}_{[1,3]} = \dfrac{f(3) - f(1)}{3 - 1} = \dfrac{12}{2} = 6$

$3c^2 - 7 = 6 \qquad 3c^2 = 13$

$c^2 = \frac{13}{3} \qquad c = \pm\sqrt{\frac{13}{3}}$ $\qquad \cancel{c = -\sqrt{\frac{13}{3}}}$ out of domain

$\therefore c = \sqrt{\frac{13}{3}} \in (1, 3)$ ✓

note: $\sqrt{\frac{3}{3}} = 1 \quad \sqrt{\frac{27}{3}} = 3$