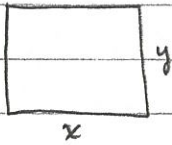


4.4 Homework Solutions

① Min Perimeter: $P = 2x + 2y$



$$P(x) = 2x + 2\left(\frac{16}{x}\right)$$

$$P(x) = 2x + \frac{32}{x}$$

$$P'(x) = 2 - \frac{32}{x^2} = 0 \quad \frac{32}{x^2} = 2 \quad x^2 = \frac{32}{2} = 16$$

$$16 = xy$$

$$y = \frac{16}{x}$$

$$x = \sqrt{16} \quad x = 4$$

out

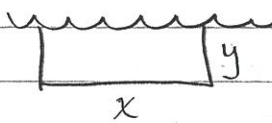
$$P''(x) = \frac{64}{x^3} \quad P''(4) > 0 \therefore P \text{ is min at } x=4$$

~~D: $0 < x < 16$~~

D: $0 < x < \infty$ Answer: $P = 2(4) + 2(4) = \boxed{16 \text{ cm}}$

$$x = 4 \text{ cm} \quad y = 4 \text{ cm}$$

②



MAX Area: $A = \text{base} \cdot \text{height}$

$$A = x(y) \quad D: 0 < y < 400$$

$$P = x + 2y = 800$$

$$y = 400 - \frac{x}{2}$$

or

$$x = 800 - 2y$$

$$A(y) = (800 - 2y)(y)$$

$$A(y) = 800y - 2y^2$$

$$A'(y) = 800 - 4y = 0 \quad y = 200$$

~~D: $0 < x < 400$~~

D: $0 < x < 800$

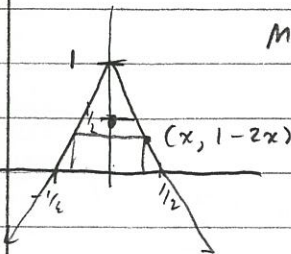
$$A''(y) = -4 \quad A''(200) < 0$$

$\therefore A$ is max at $y = 200$

Answer: $y = 200 \text{ m} \quad x = 400 \text{ m}$

MAX Area: $\boxed{8,000 \text{ m}^2}$

③



MAX Area: $A = \text{base} \cdot \text{height}$

$$A(x) = 2x(1-2x)$$

$$A(x) = 2x - 4x^2$$

$$A'(x) = 2 - 8x = 0 \quad x = \frac{1}{4}$$

$$A''(x) = -8 \quad A''(\frac{1}{4}) < 0 \therefore A \text{ is max at } x = \frac{1}{4}$$

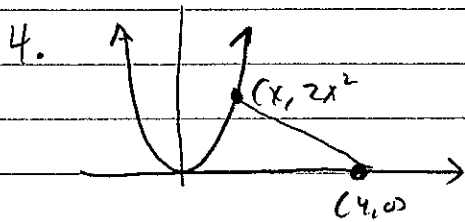
D: $0 < x < \frac{1}{2}$

$$f(x) = \begin{cases} 1-2x; & x \geq 0 \\ 1+2x; & x < 0 \end{cases}$$

$$\text{base} = 2\left(\frac{1}{4}\right) = \frac{1}{2}$$

$$\text{height} = 1 - 2\left(\frac{1}{4}\right) = \frac{1}{2}$$

$$\text{Area} = \frac{1}{4}$$



min Distance : $D = \sqrt{(x-4)^2 + (2x^2-0)^2}$

$$Q(x) = D^2(x) = (x-4)^2 + (2x^2)^2$$

$$= x^2 - 8x + 16 + 4x^4$$

$$= 4x^4 + x^2 - 8x + 16$$

$$Q'(x) = 16x^3 + 2x - 8 = 0$$

D: $x > 0$ Logically
or

$$8x^3 + x - 4 = 0$$

D: $x \in \mathbb{R}$ technically
or

$$x = 0.741$$

Q is min at $x = 0.741$ b/c

D: $(0, 18)$ logically

Q'(x) goes from negative to positive at $x = 0.741$

Answer: $(0.741, 1.098)$

5. MAX AREA : $A(x) = 2x(4 \cos(0.5x))$

D: $0 < x < \pi$ $A'(x) = 0$ at $x = 1.721$

A is max at 1.721 b/c A' goes from positive to negative at $x = 1.721$

Answer: $A(1.721) = 8.978$ units²

6. MAX AREA $A(x) = 2x \left(\frac{4.6 - x^2}{1 + x^2} \right)$

D: $0 < x < 2.145$ $A'(x) = 0$ at $x = 0.751$

A is max at $x = 0.751$ b/c

$A'(x)$ goes from positive to negative at $x = 0.751$

Answer: $A(0.751) = 3.876$ units²

7. $y = x^3 + ax^2 + bx$

$$y' = 3x^2 + 2ax + b : y'(4) = 48 + 8a + b = 0$$

$$y'' = 6x + 2a : y''(4) = 24 + 2a > 0 \checkmark$$

$$y''(1) = 6 + 2a = 0$$

$$\boxed{a = -3 \quad b = -24}$$

check

$$y''(1) = 6 + 2a = 0$$

$$\therefore a = -3$$

$$y' = 3x^2 - 6x - 24 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = -2, 4$$

$$48 - 24 + b = 0$$

$$b = -24$$

$$y'' = 6x - 6 = 0 \quad x = 1$$

$y''(4) > 0 \checkmark$
u'' check sign at $x = 1 \checkmark$