BC.Q201.L3: HOMEWORK

TECHNOLOGY SECTION

1. A function f is defined on (0.250, 1.250) and the *derivative* of the function is given by
$f'(x) = x\sin(e^{2x} - x)$ $f'(x) = 0$ at $x = 0.669, 0.992, 1.181$

Round Answer to three decimal places

A. Find the interval(s) on which f is increasing. Justify your answer.

f is increasing on $(0.250, 0.669] \cup [0.992, 1.181]$ b/c $f'(x) > 0$ on $(0.250, 0.669) \cup (0.992, 1.181)$

B. For what value(s) of x does f have a local minimum? Justify your answer.

f has a local min at $x = 0.992$ b/c $f'(x)$ goes from negative to positive at $x = 0.992$

C. On what interval is f concave upward? Justify your answer.

$f''(x) = 0$ at $x = 0.463$, $x = 0.870$, $x = 1.099$

f is concave up on $(0.250, 0.463) \cup (0.870, 1.099)$ b/c $f''(x) > 0$ on this interval.

D. How many points of inflection are there on the graph of $f(x)$? Justify your answer.

There are three points of inflection on $(0.250, 1.250)$ b/c $f''(x)$ changes sign three times on this interval.

E. Suppose we extended f to include the endpoints at $a = 0.250$ and $b = 1.250$. Use an endpoint analysis to determine whether each endpoint is a local maximum or local minimum.

There is a local <u>min</u> at $x = 0.250$

There is a local <u>min</u> at $x = 1.250$

$f(0.552) = 4$

F. Write and equation of the line tangent to $f(x)$ at $x = 0.522$.   Home: $y1(0.522)$

$y = f(0.522) + f'(0.522)(x - 0.552) \rightarrow y = 4 + 0.383(x - 0.522)$

G. Use the tangent line in part (F) to estimate $f(0.3)$

$f(0.3) \approx L(0.3) = 4 + 0.383(0.3 - 0.552)$

$= \boxed{3.915}$

NON TECHNOLOGY SECTION

2. Let $y = 4x^3 + 21x^2 + 36x - 20$

A. On what interval is the graph of y concave downward? Justify your answer.

$y' = 12x^2 + 42x + 36$ $\quad y'' = 24x + 42 = 0 \quad x = -7/4$

$- - - - - - \; -7/4 \; + + + + + \quad y''$ (x)

y is concave downward on $(-\infty, -7/4)$ b/c $y'' < 0$ on this interval.

B. The graph of y has how many points of inflection? Justify your answer.

y has one point of inflection b/c y'' changes sign one time.

3. Let $f(x) = 3x - x^3 + 5$

Use the second derivative test to determine the local extremes of $f(x)$.

$f'(x) = 3 - 3x^2 = 0 \quad x = \pm 1$

$f''(x) = -6x \qquad f''(1) = -6 < 0 \quad \therefore$ f has a local max at $x = 1$

* So local max is $f(1) = \boxed{7}$

$f''(-1) = 6 > 0 \quad \therefore$ f has a local min at $x = -1$

4. Let $f(x) = xe^x$

* so local min is $f(-1) = \boxed{3}$

Use the second derivative test to determine the local extremes of $f(x)$.

$f'(x) = xe^x + e^x \leftarrow$ Product Rule $\quad xe^x + e^x = 0 \quad e^x(x+1) = 0 \quad e^x \neq 0 \; \boxed{x = -1}$

$f''(x) = xe^x + e^x + e^x \quad f''(-1) = (-1)e^{-1} + e^{-1} + e^{-1} = e^{-1}(2-1) = e^{-1} > 0$

$\therefore$ f has a local min at $x = -1$ * so $f(-1) = \boxed{-e^{-1}}$ is the local min

5. Let $f'(x) = (x-1)^2(x-2)$ be the derivative of the function of $y = f(x)$.

A. Find the x-value(s) where the function f has a local minimum. Justify your answer.

$f'(x) = 0$ at $x = 1, 2$

$- - - - \; 1 \; - - - \; 2 \; + + + \quad f'(x)$ (x)

Option 1: f has a local min at $x = 2$ b/c $f'(x)$ changes from negative to positive at $x = 2$.

OR

Option 2: $y''(1) = 0 \rightarrow$ neither max or min at $x = 1$

$y''(2) > 0 \rightarrow \therefore$ f has a local min at $x = 2$.

B. Find the x-value(s) where the function f has a point of inflection. Justify your answer.

$f''(x) = (x-1)^2 + (x-2) \cdot 2(x-1)$

$= (x-1)[(x-1) + 2(x-2)]$

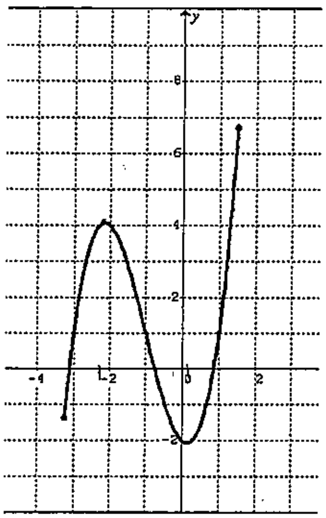
$= (x-1)[3x-5] = 0 \rightarrow x = 1, 5/3$

$+ + + \; 1 \; - - - - \; 5/3 \; + + + + \quad f''(x)$ (x)

f has a point of inflection at $x = 1, 5/3$ b/c f'' changes sign at these x-values.

GRAPH ANALYSIS SECTION

6. Consider the function $y = f(x)$ shown below.



A. Estimate the x value(s) where $f'(x) = 0$.

$x = -2.25 \quad x = 0.25$

B. Estimate the x value(s) where $f'(x) > 0$.

$x \in (-3.25, -2.25) \cup (0.25, 1.5)$

C. Estimate the x value(s) where $f'(x) < 0$.

$x \in (-2.25, 0.25)$

D. Estimate the x value(s) where $f''(x) = 0$.

$x = -1$

E. Estimate the x value(s) where $f''(x) > 0$.

$x \in (-1, 1.5)$

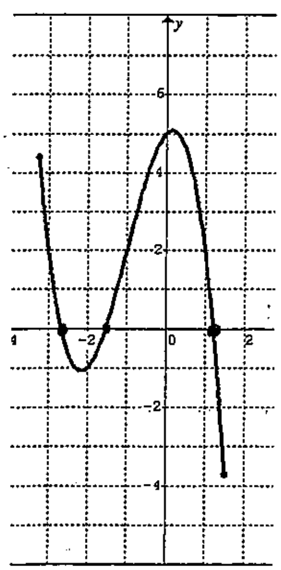
F. Estimate the x value(s) where $f''(x) < 0$.

$x \in (-3.25, -1)$

G. Estimate the x value(s) where $f'(x)$ is decreasing. Same as $f''(x) < 0$

$x \in (-3.25, -1)$

7. Consider the **derivative** graph function $y = f'(x)$ shown below.



A. Estimate the x-value(s) where f is increasing.

$x \in [-3.25, -2.75] \cup [-1.5, 1.25]$

B. Estimate the x-value(s) where f is decreasing.

$x \in [-2.75, -1.5] \cup [1.25, 1.5]$

C. Estimate the x-value(s) where f is concave up. ($f'(x)$ increasing)

$x \in (-2.25, 0.25)$

D. Estimate the x-value(s) where f is concave down. ($f'(x)$ dec.)

$x \in (-3.25, -2.25) \cup (0.25, 1.5)$