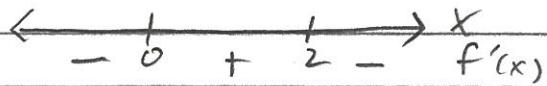


BC Q201 Lesson 2 • HW Solutions

AB Q203. Lesson 2 . HW. Solutions.

1. $f(x) = -2x^3 + 6x^2 - 3$

$$f'(x) = -6x^2 + 12x = 6x(2-x) = 0 \quad x = 0, 2$$



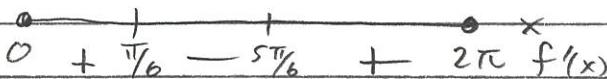
a) f is decreasing on $(-\infty, 0] \cup [2, \infty)$ because $f'(x) < 0$
on $(-\infty, 0) \cup (2, \infty)$

b) f has a local min at $x=0$ b/c $f'(x)$ goes from neg to pos^{itive} at $x=0$.
 f has a local max at $x=2$ b/c $f'(x)$ goes from pos to neg^{ative} at $x=2$.

c) $f(0) = -3$ is the local min
 $f(2) = 5$ is the local max

2. $f(x) = x + 2 \cos(x) \quad [0, 2\pi]$

$$f'(x) = 1 - 2\sin x = 0 \rightarrow \sin x = \frac{1}{2} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$



a) f is increasing on $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$

b/c $f'(x) > 0$ on $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$ * *Brackets or () about 0 and 2π

b) f has a local max at $x = \frac{\pi}{6}$ b/c $f'(x)$ goes from pos to neg^{ative}
 f has a local min at $x = \frac{5\pi}{6}$ b/c $f'(x)$ goes from neg^{ative} to pos^{itive} at $x = \frac{5\pi}{6}$

c) $f(0)$ is a local min (increases away from start point)

$f(2\pi)$ is a local max (increases to end point)

"endpoint analysis"

d) $f(0) = 2$ $2\pi + 2$ is the absolute max and

$f(\frac{5\pi}{6}) = 2\pi + 2 - \sqrt{3}$ is the absolute min by

$f(\frac{\pi}{6}) = \frac{\pi}{6} + \sqrt{3}$ the closed interval test.

$f(\frac{5\pi}{6}) = \frac{5\pi}{6} - \sqrt{3}$

- 4.
- a. Graph
 - b. $f'(x) = 0$ at $x = -0.507, x = 0.494$
 - c. f is increasing on $(-1, -0.507] \cup [0.494, 1)$
because $f'(x) > 0$ on $(-1, -0.507) \cup (0.494, 1)$
 - d. f has a local max at $x = -0.507$
because $f'(x)$ goes from positive to negative at $x = -0.507$

- 5.
- a. Graph
 - b. $f'(x) = 0$ at $x = -1.794, 1.794$
 - c. f is increasing on $[-1.794, 1.794]$ b/c
 $f'(x) > 0$ on $(-1.794, 1.794)$
 - d. f has a local max at $x = 1.794$ b/c
 $f'(x)$ goes from positive to negative at 1.794

- 6.
- a. Graph
 - b. $f'(x) = 0$ at $x = -1.253, 0, 1.253$
 - c. f is increasing on $[-2, -1.253] \cup [0, 1.253]$
b/c $f'(x) > 0$ on $(-2, -1.253) \cup (0, 1.253)$
 - d. f has a local min at $x = 0$ b/c
 $f'(x)$ goes from negative to positive at $x = 0$

f has a local min at $x = -2$ b/c f increases away from $f(-2)$
 f has a local min at $x = 2$ b/c f decreases to $f(2)$

1990 AB5

(a) $f(x) = 0$

$$\sin^2 x - \sin x = 0$$

$$\sin x(\sin x - 1) = 0 \rightarrow x = 0, \pi, \frac{\pi}{2}$$

$$\sin x = 0 \text{ or } \sin x = 1$$

(b) $f'(x) = 2\sin x \cos x - \cos x = 0$

$$\cos x(2\sin x - 1) = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\cos x = 0 \text{ or } \sin x = \frac{1}{2}$$

f is increasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right] \cup \left[\frac{5\pi}{6}, \frac{3\pi}{2}\right]$

b/c $f'(x) > 0$ on $(\frac{\pi}{6}, \frac{\pi}{2}) \cup (\frac{5\pi}{6}, \frac{3\pi}{2})$.

(c) interior critical pts: $x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

endpoint critical pts: $x = 0, 3\pi/2$

$$f(0) = 0 - 0 = 0$$

The absolute max is

2

$$f\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$

The absolute min is

- $\frac{1}{4}$

$$f\left(\frac{\pi}{2}\right) = 1 - 1 = 0$$

$$f\left(\frac{5\pi}{6}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$

$$f\left(\frac{3\pi}{2}\right) = 1 + 1 = 2$$

Lesson 2 HW (additional Solutions)

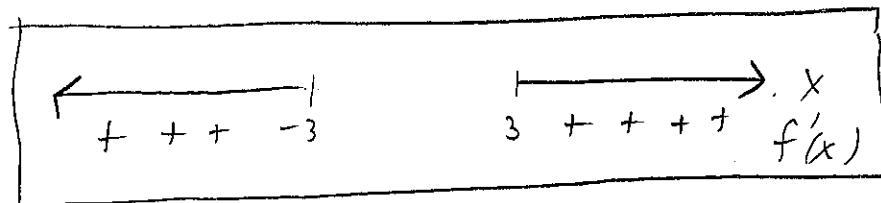
3. $f(x) = x \sqrt{x^2 - 9}$ $D: (-\infty, -3] \cup [3, \infty)$

$$f'(x) = x \cdot \frac{1}{2}(x^2 - 9)^{-\frac{1}{2}} \cdot 2x + \sqrt{x^2 - 9}$$

$$= \frac{x^2}{\sqrt{x^2 - 9}} + \sqrt{x^2 - 9} = \frac{x^2}{\sqrt{x^2 - 9}} + \frac{x^2 - 9}{\sqrt{x^2 - 9}}$$

$$= \frac{2x^2 - 9}{\sqrt{x^2 - 9}}$$

$f'(x) = 0 \quad x = \pm \frac{3}{\sqrt{2}} \rightarrow$ out of domain
 $f'(x)$ DNE when $x = \pm 3$



f is never decreasing because $f'(x)$ is never negative.

f has a local max at $x = -3$ b/c f increases up to $f(-3)$.

f has a local min at $x = 3$ b/c f increases away from $f(+3)$.

positive 3