BC Q201 Lesson 2. HW Solutions

AB Q203. Lesson 2. HW. Solutions.

1. $f(x) = -2x^3 + 6x^2 - 3$

$f'(x) = -6x^2 + 12x = 6x(2-x) = 0 \qquad x = 0, 2$

x: $\quad -\ \ 0\ \ +\ \ 2\ \ -\quad f'(x)$

a) f is decreasing on $(-\infty, 0] \cup [2, \infty)$ because $f'(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$

b) f has a local min at $x = 0$ b/c $f'(x)$ goes from negative to positive at $x = 0$.
f has a local max at $x = 2$ b/c $f'(x)$ goes from positive to negative at $x = 2$.

c) $f(0) = -3$ is the local min
$f(2) = 5$ is the local max

2. $f(x) = x + 2\cos(x) \quad [0, 2\pi]$

$f'(x) = 1 - 2\sin x = 0 \rightarrow \sin x = \frac{1}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

x: $\quad 0\ \ +\ \ \frac{\pi}{6}\ \ -\ \ \frac{5\pi}{6}\ \ +\ \ 2\pi\quad f'(x)$

a) f is increasing on $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$
b/c $f'(x) > 0$ on* $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$* *Bracket or ( ) about 0 and 2π

b) f has a local max at $x = \frac{\pi}{6}$ b/c $f'(x)$ goes from positive to negative at $x = \frac{\pi}{6}$
f has a local min at $x = \frac{5\pi}{6}$ b/c $f'(x)$ goes from negative to positive at $x = \frac{5\pi}{6}$

c) $f(0)$ is a local min (increases away from start point)
$f(2\pi)$ is a local max (increases to end point)
"endpoint analysis"

d) $f(0) = 2$
$f(2\pi) = 2\pi + 2$
$f(\frac{\pi}{6}) = \frac{\pi}{6} + \sqrt{3}$
$f(\frac{5\pi}{6}) = \frac{5\pi}{6} - \sqrt{3}$

$2\pi + 2$ is the absolute max and $\frac{5\pi}{6} - \sqrt{3}$ is the absolute min by the closed interval test.

4. a. Graph

b. $f'(x) = 0$ at $x = -0.507$, $x = 0.494$

c. f is increasing on $(-1, -0.507] \cup [0.494, 1)$
because $f'(x) > 0$ on $(-1, -0.507) \cup (0.494, 1)$

D. f has a local max at $x = -0.507$
because $f'(x)$ goes from positive to negative at $x = -0.507$

5. a. Graph

b. $f'(x) = 0$ at $x = -1.794, 1.794$

c. f is increasing on $[-1.794, 1.794]$ b/c
$f'(x) > 0$ on $(-1.794, 1.794)$

d. f has a local max at $x = 1.794$ b/c
$f'(x)$ goes from positive to negative at 1.794

6. a. Graph

b. $f'(x) = 0$ at $x = -1.253, 0, 1.253$

c. f is increasing on $[-2, -1.253] \cup [0, 1.253]$
b/c $f'(x) > 0$ on $(-2, -1.253) \cup (0, 1.253)$

d. f has a local min at $x = 0$ b/c
$f'(x)$ goes from negative to positive at $x = 0$

f has a local min at $x = -2$ b/c f increases away from $f(-2)$
f has a local min at $x = 2$ b/c f decreases to $f(2)$

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(a) $f(x) = 0$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0 \rightarrow x = 0, \pi, \frac{\pi}{2}$

$\sin x =$ or $\sin x = 1$

(b) $f'(x) = 2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

f is increasing on $[\pi/6, \pi/2] \cup [5\pi/6, \frac{3\pi}{2}]$
b/c $f'(x) > 0$ on $(\pi/6, \pi/2) \cup (5\pi/6, 3\pi/2)$.

(c) interior critical pts: $x = \pi/2, \frac{\pi}{6}, \frac{5\pi}{6}$
endpoint critical pts: $x = 0, 3\pi/2$

$f(0) = 0 - 0 = 0$
$f(\pi/6) = (1/2)^2 - \frac{1}{2} = -1/4$
$f(\pi/2) = 1 - 1 = 0$
$f(5\pi/6) = (1/2)^2 - 1/2 = -1/4$
$f(3\pi/2) = 1 + 1 = 2$

The absolute max is $\boxed{2}$

The absolute min is $\boxed{-1/4}$

Lesson 2 HW (additional Solutions)

3. $f(x) = x\sqrt{x^2 - 9}$ $\qquad D: (-\infty, -3] \cup [3, \infty)$

$$f'(x) = x \cdot \frac{1}{2}(x^2 - 9)^{-1/2} \cdot 2x + \sqrt{x^2 - 9}$$

$$= \frac{x^2}{\sqrt{x^2-9}} + \sqrt{x^2 - 9} = \frac{x^2}{\sqrt{x^2-9}} + \frac{x^2 - 9}{\sqrt{x^2 - 9}}$$

$$= \frac{2x^2 - 9}{\sqrt{x^2 - 9}}$$

$f'(x) = 0 \quad x = \pm \frac{3}{\sqrt{2}} \rightarrow$ out of domain

$f'(x)$ DNE when $x = \pm 3$

$\leftarrow$ + + + -3	3 + + + + $\rightarrow$ x
	$f'(x)$

f is never decreasing because $f'(x)$ is never negative.

f has a local max at $x = -3$ b/c f increases up to $f(-3)$.

f has a local min at $x = 3$ b/c f increases away from $f(+3)$.

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