Lesson 2 3.8, 3.9 HW SOLUTIONS

#43 $y = (\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x)$

$\frac{dy}{dx} = (\sin x)^x \left[x \frac{\cos x}{\sin x} + \ln(\sin x) \right]$

or

$\frac{dy}{dx} = (\sin x)^x \left[x \cot x + \ln(\sin x) \right]$

#45 $y = \sqrt[5]{\text{stuff}}$

$\ln y = \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5) \right]$

$\frac{1}{y}\frac{dy}{dx} = \frac{4}{5} \cdot \frac{1}{x-3} + \frac{1}{5}\frac{1}{x^2+1}(2x) - \frac{3}{5}\frac{1}{2x+5} \cdot 2$

$\frac{dy}{dx} = \sqrt[5]{\text{stuff}} \left[\frac{4}{5}\frac{1}{x-3} + \frac{2}{5}\frac{x}{x^2+1} - \frac{6}{5}\frac{1}{2x+5} \right]$

#46 $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

$\ln y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{2}{3}\frac{1}{x+1}$

$\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3}\frac{1}{x+1} \right]$

#47 $y = x^{\ln x}$

$\ln y = \ln x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \frac{1}{x}$

$\frac{dy}{dx} = x^{\ln x}\left[\frac{2\ln x}{x}\right]$

MC #2

$\frac{dy}{dx} = 3\cos^2(3x-2)(-\sin(3x-2))\cdot 3$

$= -9\cos^2(3x-2)\sin(3x-2)$

$\boxed{A}$

MC #3

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}}\cdot 2 = \frac{2}{\sqrt{1-4x^2}}$

$\boxed{C}$

#28 A] $f'(x) = 5x^4 + 6x^2 + 1 > 0 \quad \forall x$

$\therefore$ f is always increasing

$\therefore$ f is a one-to-one function

$\therefore$ The inverse of f is also a function

B] $f(1) = 3 \quad f'(1) = 12$

C] $f^{-1}(3) = 1 \quad (f^{-1})'(3) = \frac{1}{f'(1)} = \frac{1}{12}$

#29 A] $f'(x) = -\sin x + 3 > 0 \quad \forall x$

since $-1 \le -\sin x \le 1$

$2 \le -\sin x + 3 \le 4$

$\therefore$ f is always increasing

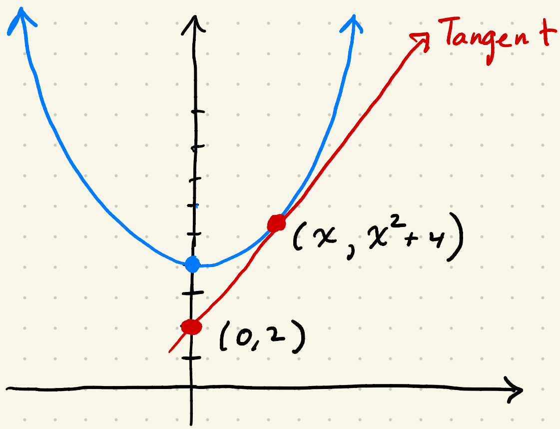
$\therefore$ f is one to one function

$\therefore$ The inverse of f is also a function

B] $g'(1) = \frac{1}{f'(0)} = \frac{1}{-\sin(0)+3} = \frac{1}{3}$

$\cos a + 3a = 1 \rightarrow$ This happens at $a=0$

Extra Problem $f(x) = x^2 + 4$



$$\boxed{m} = \frac{\Delta Y}{\Delta X} = \frac{(x^2+4)-(2)}{x-0} = \frac{x^2+2}{x}$$

$$\boxed{m} = f'(x) = 2x$$

$$\frac{x^2+2}{x} = 2x$$

$$2x^2 = x^2 + 2$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

question calls for positive x-value

$$\boxed{x = +\sqrt{2}}$$

CHECK

$$m\Big|_{\sqrt{2}} = \frac{(\sqrt{2})^2 + 2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \checkmark$$

$$f'(\sqrt{2}) = 2\sqrt{2} \checkmark$$