3.8 3.9 Lesson 1

HW Solutions

1. $y = \cos^{-1}(x^2)$ $\quad \frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x$

3. $y = \sin^{-1}(\sqrt{2}\,t)$ $\quad y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2}$

5. $y = \sin^{-1}\left(\frac{3}{t^2}\right)$ $\quad y' = \boxed{\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2} \cdot -6t^{-3}}}$ $\quad \frac{1}{\sqrt{1-\frac{9}{t^4}}} \cdot \frac{-6}{t^3} = \frac{1}{\sqrt{\frac{t^4-9}{t^4}}} \cdot \frac{-6}{t^3}$

$= \frac{-6}{t\sqrt{t^4-9}}$

7. $y = x\sin^{-1}x + \sqrt{1-x^2}$

$\frac{dy}{dx} = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

9. $v(t) = x'(t) = \frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}} \cdot \frac{1}{4}$ $\quad x'(3) = \frac{1}{\sqrt{1-\frac{9}{16}}} \cdot \frac{1}{4} = \frac{4}{\sqrt{7}} \cdot \frac{1}{4} = \boxed{\frac{1}{\sqrt{7}}}$

11. $x'(t) = \frac{1}{1+t^2}$ $\quad x'(2) = \frac{1}{1+4} = \frac{1}{5}$

13. $\frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot 2$

15. $\frac{dy}{dx} = \frac{-1}{(x^2+1)\sqrt{(x^2+1)^2-1}} \cdot 2x$

17. $\frac{dy}{dx} = \boxed{\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}} \cdot \frac{-1}{t^2}} = \frac{-1}{\sqrt{\frac{1}{t^2}-1}\,t} = \frac{-1}{\sqrt{1-t^2}}$

19. $y' = \boxed{\frac{-1}{1+(\sqrt{t-1})^2} \cdot \frac{1}{2\sqrt{t-1}}} = \frac{-1}{1+t-1} \cdot \frac{1}{2\sqrt{t-1}} = \frac{-1}{2t\sqrt{t-1}}$

21. $y' = \boxed{\frac{1}{1+\left(\sqrt{x^2-1}\right)^2} \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x - \frac{1}{|x|\sqrt{x^2-1}}}$

$\frac{1}{1+x^2-1} \cdot \frac{1}{\sqrt{x^2-1}} \cdot x - \frac{1}{x\sqrt{x^2-1}}$

$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = 0$ [interesting]

#35 TRUE

#36. FALSE

#37. $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$

$= \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2}$

$= \frac{1}{\sqrt{\frac{4}{4}-\frac{x^2}{4}}} \cdot \frac{1}{2} = \frac{1}{\frac{\sqrt{4-x^2}}{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{4-x^2}}$ (E)

#38. $\frac{d}{dx}\tan^{-1}(3x) = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$ (D)

#39. $\frac{d}{dx}\sec^{-1}(x^2) = \frac{1}{x^2\sqrt{x^4-1}} \cdot 2x = \frac{2}{x\sqrt{x^4-1}}$ (A)

#40. $y' = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$ $\quad y'\Big|_{x=1} = \frac{2}{1+4} = \frac{2}{5}$ (C)

5. $y' = e^{2x/3} \cdot \frac{2}{3}$

7. $y' = e^2 - e^x$

9. $y' = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

11. $y' = 8^x \ln 8$

13. $y' = 3^{\csc x} \ln 3 \cdot (-\csc x \cot x)$

15. $y' = \frac{1}{x^2} \cdot 2x$

17. $y' = \boxed{\frac{1}{1/x} \cdot \frac{-1}{x^2}} = -\frac{1}{x} \quad x > 0$

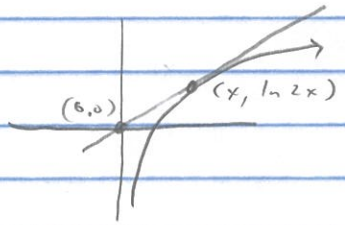
19. $y' = \frac{1}{\ln x} \cdot \frac{1}{x}$

21. $y = \frac{\ln x^2}{\ln 4} \qquad y' = \frac{1}{\ln 4} \cdot \frac{1}{x^2} \cdot 2x$

23. $y = \frac{\ln(\frac{1}{x})}{\ln(2)} \qquad y' = \frac{1}{\ln(2)} \cdot \frac{1}{1/x} \cdot \frac{-1}{x^2} \qquad x > 0$

25. $y' = \ln 2 \cdot \frac{1}{x} \cdot \frac{1}{\ln 2} \qquad x > 0$

31.



$m = \frac{\Delta y}{\Delta x} = \frac{\ln 2x}{x}$

$m = \frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

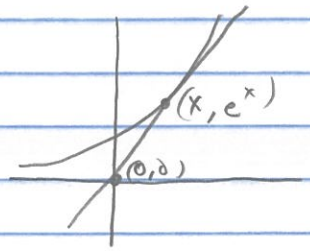
$\frac{\ln 2x}{x} = \frac{1}{x} \quad \therefore \ln 2x = 1$

$2x = e$

$x = e/2$

$m = \frac{1}{x} = \frac{1}{e/2} = \boxed{\frac{2}{e}}$

49.



$m = \frac{\Delta y}{\Delta x} = \frac{e^x}{x} \qquad \frac{e^x}{x} = e^x$

$m = \frac{dy}{dx} = e^x \qquad xe^x - e^x = 0$

$e^x(x-1) = 0 \quad \therefore x = 1$

$m = e$ $\boxed{y - e = e(x-1)}$ or $\boxed{y = ex}$

LESSON 1 HW EXTENTION
SIMPLIFYING EXPRESSIONS

3.8 #5

$$\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}} \cdot -6t^{-3} = \frac{1}{\sqrt{1-\frac{9}{t^4}}} \cdot \frac{-6}{t^3}$$

$$= \frac{1}{\sqrt{\frac{t^4}{t^4}-\frac{9}{t^4}}} \cdot \frac{-6}{t^3} = \frac{1}{\sqrt{\frac{t^4-9}{t^4}}} \cdot \frac{-6}{t^3}$$

$$= \frac{1}{\frac{\sqrt{t^4-9}}{t^2}} \frac{-6}{t^3} = \boxed{\frac{-6}{t\sqrt{t^4-9}}}$$

3.8 #17

$$\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}} \cdot \frac{-1}{t^2} = \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}-1}} \frac{-1}{t^2}$$

$0 < t < 1$

$$= \frac{-1}{t\sqrt{\frac{1-t^2}{t^2}}} = \frac{-1}{t\frac{\sqrt{1-t^2}}{t}} = \boxed{\frac{-1}{\sqrt{1-t^2}}}$$

3.8 # 37 $\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{1}{\frac{\sqrt{4-x^2}}{4}} \cdot \frac{1}{2} = \frac{1}{\frac{\sqrt{4-x^2}}{2}} \cdot 2$

$$= \boxed{\frac{1}{\sqrt{4-x^2}}}$$