BC. Q103 L3 (4.5) HW Solutions

1. $f(x) = 2 + \sin x \qquad a = 0$

$f'(x) = \cos x \qquad L(x) = 2 + x$

$f'(0) = 1 \qquad f(-0.1) \approx L(-0.1) = 2 - (0.1) = \boxed{1.9}$

$f(0) = 2$

2. $f(x) = x^3 - 2x + 3 \qquad a = 2$

$f'(x) = 3x^2 - 2 \qquad L(x) = 7 + 10(x-2)$

$f'(2) = 10 \qquad f(1.8) \approx L(1.8) = 7 + 10(-0.2) = \boxed{5}$

$f(2) = 7$

3. $f'(x) = \sqrt{1+x} \qquad a = 3$

$f'(3) = 2 \qquad L(x) = 7 + 2(x-3)$

$f(3) = 7 \qquad f(2.5) \approx L(2.5) = 7 + 2(-0.5) = \boxed{6}$

4. $f'(x) = \cos(x^2) \qquad a = 0$

$f'(0) = 1 \qquad L(x) = 4 + x$

$f(0) = 4 \qquad f(0.4) \approx L(0.4) = 4 + 0.4 = \boxed{4.4}$

5. $f'(x) = 2 - x \qquad a = 4$

$f'(4) = -2 \qquad L(x) = 3 - 2(x-4)$

$f(4) = 3 \qquad f(4.2) \approx L(4.2) = 3 - 2(0.2) = \boxed{2.6}$

6. $\Delta f \approx df = f'(a)\,dx$

$= (2(2)-5)(0.1)$

$= (-1)(0.1)$

$= \boxed{-0.1}$

$a = 2 \quad dx = 0.1$

$f'(x) = 2x - 5$

7. $\Delta V \approx dV = V'(a)\,dx$

$= V'(9)(-0.01)$

$= 3(81)(-0.01)$

$= \boxed{-2.43}\ mm^3$

$a = 9 \quad dx = -0.01$

$V = x^3$

$V'(x) = 3x^2$

81
3
243

8. $\Delta C \approx dC = C'(a)\,dd$

$= \pi(0.02)$

$= \boxed{0.02\pi}$ in

$a = 2 \quad dx = 0.02$

$C = \pi d$

$C'(d) = \pi$

2
27
3
81

9. $\sqrt[3]{26}$

Let $f(x) = \sqrt[3]{x} \qquad a = 27$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$f'(27) = \frac{1}{3(27)^{2/3}} = \frac{1}{27}$

$f(27) = \sqrt[3]{27} = 3$

$L(x) = 3 + \frac{1}{27}(x-27)$

9

$\sqrt[3]{26} = f(26) \approx L(26)$

$= 3 + \frac{1}{27}(-1)$

$= 3 - \frac{1}{27}$

$= \frac{81}{27} - \frac{1}{27}$

$= \boxed{\frac{80}{27}}$ or $2\frac{26}{27}$

1987 #5

a) $V = \frac{1}{2} b \cdot h \cdot l = \frac{1}{2}(2)(3)(5) = 15 \text{ ft}^3$

b) Given $\frac{dV}{dt} = -2$

Find $\frac{dh}{dt}$ when $V = \frac{15}{4}$

RELATIONSHIP $V = \frac{1}{2} b \cdot h \cdot 5 = \frac{5}{2} b \cdot h$

$\frac{b}{h} = \frac{2}{3} \rightarrow b = \frac{2h}{3}$

UPDATE $V = \frac{5}{3} h^2$ $\quad V = \frac{15}{4} \quad h^2 = \frac{9}{4} \quad h = \frac{3}{2}$

$\frac{d}{dt}\left[V = \frac{5}{3} h^2\right]$

$\frac{dV}{dt} = \frac{10}{3} h \frac{dh}{dt}$

$-2 = \frac{10}{3}\left(\frac{3}{2}\right)\frac{dh}{dt}$ $\rightarrow$ $\boxed{\frac{dh}{dt} = \frac{-2}{5} \text{ ft/min}}$

c) Given $\frac{dh}{dt} = \frac{-2}{5}$ (from part b)

Find $\frac{dA}{dt}$ when $V = \frac{15}{4}$ or $h = \frac{3}{2}$

Relationship $A = 5 \cdot b$

UPDATE $A = \frac{10}{3} h$

$\frac{dA}{dt} = \frac{10}{3} \frac{dh}{dt} = \frac{10}{3}\left(\frac{-2}{5}\right) = \frac{-4}{3} \text{ ft}^2/\text{min}$

1990 AB4

Given $\frac{dr}{dt} = 0.04 = \frac{4}{100} = \frac{1}{25}$ cm/sec

A] Find $\frac{dV}{dt}$ when $r = 10$

Rel $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

I/O $\frac{dV}{dt} = 4\pi (10)^2 \left(\frac{1}{25}\right) = \underline{16\pi}$ cm^3/sec

B] Find $\frac{dA}{dt}$ when $V = 36\pi \rightarrow 36\pi = \frac{4}{3}\pi r^3$

$r^3 = 27 \rightarrow r = 3$

Rel $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

I/O $\frac{dA}{dt} = 2\pi (3)\left(\frac{1}{25}\right) = \underline{\frac{6\pi}{25}}$ cm^2/sec

C] Find r when $\frac{dV}{dt} = \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dr}{dt}$

$4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}$

$4\pi r^2 = 1$

$r^2 = \frac{1}{4\pi}$

$r = \sqrt{\frac{1}{4\pi}}$ cm