BC. LESSON 2 HW. Solutions

1 4.6 #11

a) Given $\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$

Find $\frac{dr}{dt}$ when $r = 5$ ft

Rel $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

III D $100\pi = 4\pi(5)^2 \frac{dr}{dt}$

$\therefore \boxed{\frac{dr}{dt} = 1 \text{ ft/min}}$

b) Given $\frac{dr}{dt} = 1$ ft/min

Find $\frac{dA}{dt}$ when $r = 5$ ft

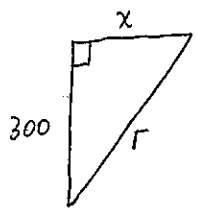
Rel $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

III D $\frac{dA}{dt} = 8\pi(5)(1) = 40\pi \text{ ft}^2/\text{min}$

$\boxed{\frac{dA}{dt} = 40\pi \text{ ft}^2/\text{min}}$

EXTRA 4.6 #14

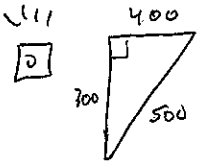



Given $\frac{dx}{dt} = 25$ ft/sec

Find $\frac{dr}{dt}$ when $r = 500$ ft

Rel $300^2 + r^2 = x^2$

III D



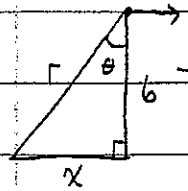

$$2r\frac{dr}{dt} = 2x\frac{dx}{dt}$$

III D $2(500)\frac{dr}{dt} = 2(400)(25)$

$\boxed{\frac{dr}{dt} = 20 \text{ ft/sec}}$

# HW SOLUTIONS

#21. Given $\frac{dr}{dt} = -2$ ft/sec

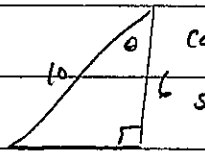



A] Find $\frac{dx}{dt}$ when $r = 10$

→ Relationship $x^2 + 36 = r^2$

$2x\frac{dx}{dt} = 2r\frac{dr}{dt}$

SNAP SHOT




$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\sec\theta = \frac{5}{3}$

$2(8)\frac{dx}{dt} = 2(10)(-2)$

$\frac{dx}{dt} = \frac{-5}{2}$ ft/sec

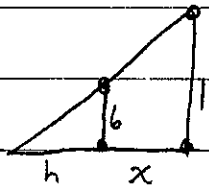
B] Find $\frac{d\theta}{dt}$ when $\frac{dx}{dt} = \frac{-5}{2}$ ft/sec

→ Relationship $\tan\theta = \frac{x}{6}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{6}\cdot\frac{dx}{dt}$

$\left(\frac{5}{3}\right)^2\frac{d\theta}{dt} = \frac{1}{6}\left(\frac{-5}{2}\right)$ $\quad \frac{d\theta}{dt} = \frac{-5}{12}\cdot\frac{9}{25} = \frac{-3}{20}$ radians/sec

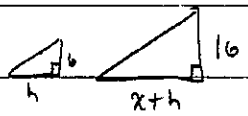
#29




Given $\frac{dx}{dt} = -5$ ft/sec

Find $\frac{dh}{dt}$

Relationship $\frac{16}{x+h} = \frac{6}{h}$ Similar triangles




$16h = 6x + 6h$

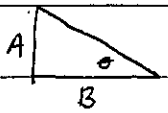
$16\frac{dh}{dt} = 6\frac{dx}{dt} + 6\frac{dh}{dt}$

$10\frac{dh}{dt} = 6\frac{dx}{dt}$

$10\left(\frac{dh}{dt}\right) = 6(-5)$

$\frac{dh}{dt} = -3$ ft/sec

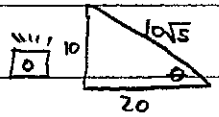
#34




Given $\frac{dA}{dt} = -2$ $\quad \frac{dB}{dt} = 1$

Find $\frac{d\theta}{dt}$ when $A = 10$ $\quad B = 20$

Rel $\tan\theta = \frac{A}{B}$ $\quad \sec^2\theta\frac{d\theta}{dt} = \frac{B\left(\frac{dA}{dt}\right) - A\left(\frac{dB}{dt}\right)}{B^2}$




$\left(\frac{\sqrt{5}}{2}\right)^2\frac{d\theta}{dt} = \frac{20(-2) - 10(1)}{(20)^2}$

$\cos\theta = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$ $\quad \frac{d\theta}{dt} = \frac{-1}{10}$ radians/sec

$\sec\theta = \frac{\sqrt{5}}{2}$

Convert after

$\frac{-1\text{ rad}}{10\text{ sec}}\cdot\frac{180\text{ deg}}{\pi\text{ rad}} = \frac{-18}{\pi}$ deg/sec

∴ I will ask for rad/sec not deg/sec

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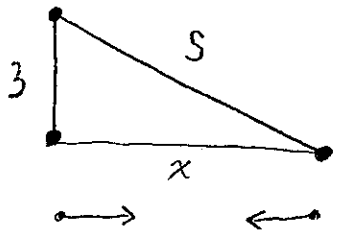
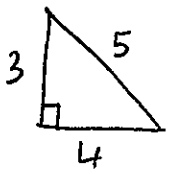



Given $\frac{dS}{dt} = -160$ mph when $S = 5$ miles

Find First find $\frac{dx}{dt}$ when $S = 5$ miles

Rel $x^2 + 9 = S^2$




$$2x\frac{dx}{dt} = 2S\frac{ds}{dt}$$

"1" $2(4)\frac{dx}{dt} = 2(5)(-160)$

$$\frac{dx}{dt} = -200$$

$x$ is getting smaller at a rate of 200 mph. The plane is contributing 120 mph to this. Therefore the car is contributing 80 mph to the distance collapse.

$$\frac{dx}{dt} = \frac{dc}{dt} + \frac{dp}{dt}$$

$$-200 = \frac{dc}{dt} - 120$$

$\therefore \frac{dc}{dt} = -80$ mph CAR is moving 80 mph.

4.6

EXTRA 16.

Given $\frac{dV}{dt} = 10\ m^3/min$

Find $\frac{dh}{dt}$ when $h = 4$

$h = \frac{3}{8}b$

$h = \frac{3}{8}(2r)$

$h = \frac{3}{4}r$

$r = \frac{4h}{3}$

Rel $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$* V = \frac{16\pi}{27}h^3$

$\frac{dV}{dt} = \frac{16\pi}{9}h^2\frac{dh}{dt}$

$10 = \frac{16\pi}{9}(16)\frac{dh}{dt}$

$\frac{90}{256\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{45}{128\pi}\ m/min$

$= \frac{4500}{128\pi}\ cm/min$

$\frac{dr}{dt} = \frac{4}{3}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{4}{3}\left(\frac{45}{128\pi}\right) = \frac{60}{128\pi} = \frac{15}{32\pi}\ m/min = \frac{1500}{32\pi}\ cm/m = \frac{375}{8\pi}\ cm/min$ ✓

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Given $\frac{dV}{dt} = -50\ m^3/min$

$\frac{dh}{dt}$ when $h = 5\ m$


$\frac{6}{h} = \frac{45}{r}$

$6r = 45h$

$r = \frac{15}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$V = \frac{225\pi}{12}h^3$

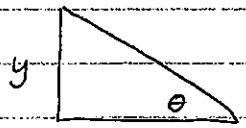
$\frac{dV}{dt} = \frac{225\pi}{4}h^2\frac{dh}{dt}$

$-50 = \frac{225\pi}{4}(25)\frac{dh}{dt}$

$\frac{-8}{225\pi} = \frac{dh}{dt} = \frac{-800}{225\pi}\ cm/min = \frac{-32}{9\pi}\ cm/min$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt} = \frac{15}{2}\left(\frac{-8}{225\pi}\right) = \frac{-60}{225\pi}\ m/min = \frac{-80}{3\pi}\ cm/m$

**EXTRA 19**

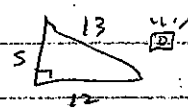



a] Given $\frac{dx}{dt} = 5$ ft/sec

Find $\frac{dy}{dt}$ when $x = 12$

Rel $x^2 + y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

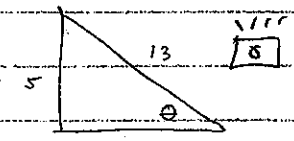



$2(12)(5) + 2(5)\frac{dy}{dt} = 0 \qquad \frac{dy}{dt} = -12$ ft/sec

b] Find $\frac{dA}{dt}$ when $y = 12$

Rel $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y\frac{1}{2}\frac{dx}{dt}$




$\frac{dA}{dt} = \frac{1}{2}(12)(-12) + \frac{5}{2}(5) = -\frac{144}{2} + \frac{25}{2} = \frac{-119}{2}$ ft²/sec

$\cos\theta = \frac{12}{13}$

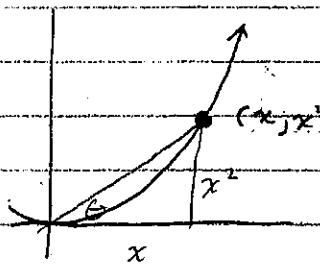
c] Rel $\sin\theta = \frac{y}{13}$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{13}\frac{dy}{dt}$

$\frac{12}{13}\frac{d\theta}{dt} = \frac{1}{13}(-12)$

$\frac{d\theta}{dt} = -1$ rad/sec

**EXTRA 25**

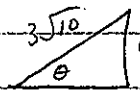



Given $\frac{dx}{dt} = 10$ m/s Find $\frac{d\theta}{dt}$ when $x = 3$

Rel $\tan\theta = \frac{x^2}{x} = x$

$\sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

9+81




$10\frac{d\theta}{dt} = 10 \qquad \frac{d\theta}{dt} = 1$ rad/sec

$\cos\theta = \frac{1}{\sqrt{10}}$

$\sec\theta = \sqrt{10}$