A.P Question

a) $xy^2 - x^3y = 6$

$$x\left(2y\frac{dy}{dx}\right) + y^2(1) - x^3\left(\frac{dy}{dx}\right) + y(-3x^2) = 0$$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3} \qquad 2xy - x^3 \neq 0$$

b) $(1)y^2 - (1)^3y = 6 \qquad y^2 - y = 6 \qquad y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, \; y = -2$

Point	$\frac{dy}{dx}$	
$(1,3)$:	$\frac{3(3)-9}{6-1} = 0$	$y = 3$
$(1,-2)$:	$\frac{3(-2)-4}{-4-1} = \frac{-10}{-5} = 2$	$y + 2 = 2(x-1)$

c) Vertical when $\frac{dy}{dx}$ DNE when $2xy - x^3 = 0$

$2xy = x^3$

$2y = x^2$

$y = x^2/2$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$\frac{x^5}{4} - \frac{x^5}{2} = 6$$

$$\frac{x^5}{4} - \frac{2x^5}{4} = 6$$

$$-\frac{x^5}{4} = 6$$

$-x^5 = 24$

$x = \sqrt[5]{-24}$

3.7 1: $x^2y + xy^2 = 6$

$\frac{d}{dx}[x^2y + xy^2 = 6]$

$x^2\frac{dy}{dx} + 2xy + x(2y)\frac{dy}{dx} + y^2(1) = 0$

$\boxed{\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2xy}}$

3.7 5: $x = \tan(y)$

$1 = \sec^2 y \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y}$

3.7 29: $y^2 = x^2 + 2x$ $\frac{dy}{dx} = \frac{x+1}{y}$

$2y\frac{dy}{dx} = 2x + 2$

$\frac{d^2y}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{y(1) - (x+1)\frac{dy}{dx}}{y^2} = \frac{y - (x+1)\left(\frac{x+1}{y}\right)}{y^2}$

3.7 49: $x^2 + xy + y^2 = 7$ $(y = 0)$

$x^2 = 7$ $x = \pm\sqrt{7}$ $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$

$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

$\left.\frac{dy}{dx}\right|_{(\sqrt{7},0)} = \frac{-2\sqrt{7}}{\sqrt{7}} = -2$ $\left.\frac{dy}{dx}\right|_{(-\sqrt{7},0)} = \frac{-2(-\sqrt{7})}{-\sqrt{7}} = -2$

3.7 #17 $x^2 + xy - y^2 = 1$

$\frac{d}{dx}[x^2 + xy - y^2 = 1]$

$2x + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 2y) = -y - 2x$

$\frac{dy}{dx} = \frac{-y-2x}{x-2y}$

$\frac{dy}{dx}\Big|_{(2,3)} = \frac{-3-4}{2-6} = \frac{-7}{-4} = 7/4$: $y - 3 = 7/4(x-2)$ T

$y - 3 = -4/7(x-2)$ N

3.7 # 21 $[6x^2 + 3xy + 2y^2 + 17y - 6 = 0]'$

$12x + 3xy' + 3y + 4yy' + 17y' = 0$

$y'(3x + 4y + 17) = -12x - 3y$

$y' = \frac{-12x - 3y}{3x + 4y + 17}$

$y'(-1, 0) = \frac{12}{-3+17} = \frac{12}{14} = \frac{6}{7}$: $y = \frac{6}{7}(x+1)$ T

$y = -\frac{7}{6}(x+1)$ N

3.7.56 ; $x^2 + 2xy - 3y^2 = 0$ $(1,1)$

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x - 6y} = \frac{x + y}{3y - x}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2}{2} = 1 \qquad m_T = 1 \quad m_N = -1$$

$y - 1 = -1(x - 1)$ $y = -x + 2$ ← NORMAL

$$x^2 + 2x(-x+2) - 3(-x+2)^2 = 0$$
$$x^2 - 2x^2 + 4x - 3x^2 + 12x - 12 = 0$$
$$-4x^2 + 16x - 12 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$

$x = 1$ ↓ already given or $x = 3$ and $y = -3 + 2 = -1$

$* P + (3, -1) *$

3.7 #57 $xy + 2x - y = 0$

$2x + y = 0$

$y = -2x$

$m = -2$

$$x\frac{dy}{dx} + y + 2 - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2-y}{x-1} = m_T$$

$$m_N = -\frac{1}{\frac{dy}{dx}} = \frac{x-1}{2+y} = -2$$

$$x - 1 = -2(2+y)$$

$$x - 1 = -4 - 2y$$

$$x = -3 - 2y$$

0: $(-3-2y)\cdot y + 2(-3-2y) - y = 0$

$$-3y - 2y^2 - 6 - 4y - y = 0$$

$$-2y^2 - 8y - 6 = 0$$

$$y^2 + 4y + 3 = 0$$

$$(y+3)(y+1) = 0 \qquad y = -1 \text{ or } y = -3$$

$y = -1$: $x = -3 + 2 = -1 \rightarrow P + (-1, -1) \rightarrow \boxed{y + 1 = -2(x+1)}$

$y = -3$: $x = -3 + 6 = 3 \rightarrow P + (3, -3) \rightarrow \boxed{y + 3 = -2(x-3)}$