

$$1. \frac{dy}{dx} = 2 \left(\frac{\sin x}{1 + \cos(2x)} \right) \cdot \left[\frac{(1 + \cos(2x)) \cdot \cos x - \sin x (-\sin(2x)) \cdot 2}{[1 + \cos(2x)]^2} \right] \quad \left[\begin{array}{l} 3.6 \\ \text{Solutions} \end{array} \right]$$

$$2. \frac{dy}{dx} = x^3 \cdot 4(2x-5)^3 \cdot 2 + (2x-5)^4 \cdot 3x^2$$

$$3. \frac{dy}{dx} = \sin^3 x \sec^2(4x) \cdot 4 + \tan(4x) \cdot 3 \sin^2 x (\cos x)$$

$$4. \frac{dy}{dx} = 3(1 + \cos^2(7x))^2 \cdot 2 \cos(7x) (-\sin(7x)) \cdot 7$$

$$5. \frac{dy}{dx} = \frac{1}{2} (\tan(5x))^{-1/2} \cdot \sec^2(5x) \cdot 5$$

$$6. \frac{dy}{dx} = 2(x - \sin(3x)) \cdot (1 - \cos(3x)) \cdot 3$$

$$7. \frac{dy}{dx} = -2(2x + \sqrt{x})^{-3} \left(2 + \frac{1}{2} x^{-1/2} \right)$$

$$8. \frac{d}{dx} \left[\frac{1}{g^2(x)} \right] = -2[g(x)]^{-3} \cdot g'(x)$$

$$9. \frac{d}{dx} [f(g(x) + x)] = f'(g(x) + x) [g'(x) + 1]$$

$$10. \frac{d}{dx} \left[\sqrt{f^2(x) + g^2(x)} \right] = \frac{1}{2} (f^2(x) + g^2(x))^{-1/2} \cdot [2f(x)f'(x) + 2g(x) \cdot g'(x)]$$

$$11. \frac{d}{dx} [f(x) \cdot g^3(x)] = f(x) \cdot 3g^2(x) \cdot g'(x) + g^3(x) \cdot f'(x)$$

$$12. \frac{d}{dx} \left[\frac{f^2(x)}{3x - g(h(x))} \right] = \frac{(3x - g(h(x))) \cdot 2f(x)f'(x) - f^2(x)[3 - g'(h(x)) \cdot h'(x)]}{[3x - g(h(x))]^2}$$

#1.

x	f	f'	g	g'	h	h'
0	2	11	-2	7	6	5
1	-4	5	5	8	-1/2	3
2	1	3	6	-1	0	4
6	-10	1/2	3	10	3/2	0

A. Find $[f(x) + 3g(x)]'$ at $x = 2$. $[f(x) + 3g(x)]' = f'(x) + 3g'(x)$

AT $x = 2$: $f'(2) + 3g'(2) = 3 + 3(-1) = \boxed{0}$

B. Find $[f(g(x))]'$ at $x = 2$. $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

AT $x = 2$: $f'(g(2)) \cdot g'(2) = f'(6) \cdot (-1)$
 $= \frac{1}{2}(-1) = \boxed{-\frac{1}{2}}$

C. Find $[f(2x - g(h(x)))]'$ at $x = 2$.

$$[f(2x - g(h(x)))]' = f'(2x - g(h(x))) \cdot [2 - g'(h(x)) \cdot h'(x)]$$

$$x = 2 : f'(4 - g(h(2))) \cdot (2 - g'(h(2)) \cdot h'(2))$$

$$f'(4 - g(0)) \cdot (2 - g'(0) \cdot 4)$$

$$f'(4 - (-2)) \cdot (2 - 7 \cdot 4)$$

$$f'(6) \cdot (-26)$$

$$\frac{1}{2} \cdot -26$$

$$\boxed{-13}$$

#2.

x	f	f'	g	g'
2	8	$\frac{1}{3}$	2	-3
3	3	2π	-4	5

A. Find $[f(g(x))]'$ at $x=2$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$\text{AT } x=2: f'(g(2)) \cdot g'(2) = f'(2) \cdot (-3) = \frac{1}{3} \cdot (-3) = \boxed{-1}$$

B. Find $[\sqrt{f(x)}]'$ at $x=2$

$$[\sqrt{f(x)}]' = \frac{1}{2} [f(x)]^{-1/2} \cdot f'(x)$$

$$\text{AT } x=2: \frac{1}{2} (f(2))^{-1/2} \cdot f'(2) = \frac{1}{2} \frac{1}{\sqrt{8}} \cdot \frac{1}{3} = \boxed{\frac{1}{6\sqrt{8}}}$$

C. Find $\left[\frac{1}{g^2(x)}\right]'$ at $x=3$

$$\left[\frac{1}{g^2(x)}\right]' = -2 [g(x)]^{-3} \cdot g'(x)$$

$$\text{AT } x=3 = -2 [g(3)]^{-3} \cdot g'(3) = -2 \frac{1}{-64} \cdot 5 = \frac{-10}{-64} = \boxed{\frac{5}{32}}$$

D. Find $[\sqrt{f^2(x)+g^2(x)}]'$ at $x=2$

$$[\sqrt{f^2(x)+g^2(x)}]' = \frac{1}{2} (f^2(x)+g^2(x))^{-1/2} \cdot [2f(x)f'(x) + 2g(x)g'(x)]$$

$$\text{AT } x=2: \frac{1}{2} (64+4)^{-1/2} \cdot [2 \cdot 8 \cdot \frac{1}{3} + 2 \cdot 2 \cdot (-3)]$$

$$= \frac{1}{2} \frac{1}{\sqrt{68}} \cdot \left[\frac{16}{3} - 12\right] = \boxed{\frac{-10}{3\sqrt{68}}}$$

#3.

x	f	f'	g	g'
0	1	5	1	1/3
1	3	-1/3	-4	-8/3

A. Find $[f(g(x))]'$ at $x=0$ $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

AT $x=0$: $f'(g(0)) \cdot g'(0) = f'(1) \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{1}{9}}$

B. Find $[(g(x)+f(x))^{-2}]'$ at $x=1$ $[(g(x)+f(x))^{-2}]' = -2(g(x)+f(x))^{-3} [g'(x)+f'(x)]$

AT $x=1$: $-2(g(1)+f(1))^{-3} [g'(1)+f'(1)]$
 $= -2(-4+3)^{-3} \left[\frac{-8}{3} + \frac{1}{3} \right] = -2(-1) \left(\frac{-9}{3} \right) = \boxed{-6}$

* (C) Find $[f(x)g^3(x)]'$ at $x=0$ $[f(x)g^3(x)]' = f(x) \cdot 3g^2(x) \cdot g'(x) + g^3(x) f'(x)$

AT $x=0$: $f(0) \cdot 3(g(0))^2 \cdot g'(0) + g^3(0) f'(0)$
 $= 1 \cdot 3 \cdot 1 \cdot \frac{1}{3} + 1 \cdot 5 = 1 + 5 = \boxed{6}$

* (D) Find $\left[\frac{f(x)}{g(x)+1} \right]'$ at $x=1$ $\left[\frac{f(x)}{g(x)+1} \right]' = \frac{(g(x)+1)f'(x) - f(x)g'(x)}{[g(x)+1]^2}$

AT $x=1$ $\frac{[g(1)+1]f'(1) - f(1)g'(1)}{[g(1)+1]^2}$
 $= \frac{-3 \cdot \frac{1}{3} - 3 \left(\frac{-8}{3} \right)}{(-3)^2} = \frac{1+8}{9} = \boxed{1}$