3.5 HW Solutions

1. $\frac{dy}{dx} = 1 + \sin x$

2. $\frac{dy}{dx} = -x^{-2} + 5\cos x$

3. $\frac{dy}{dx} = -x^2(\cos x) + \sin x(-2x) = -x^2\cos x - 2x\sin x$

4. $y = 4\sec x$ (rewritten) $\frac{dy}{dx} = 4\sec x \tan x$

5. $\frac{dy}{dx} = \frac{(1+\cot x)(-\csc^2 x) - \cot x(-\csc^2 x)}{(1+\cot x)^2}$

6. $\frac{dy}{dx} = \cos x$ $\quad \left.\frac{dy}{dx}\right|_{x=\pi} = -1$ $\quad y(\pi) = 3$

$\boxed{y-3 = -(x-\pi)}$ T

$\boxed{y-3 = +(x-\pi)}$ N

7. $\frac{dy}{dx} = x^2\cos x + \sin x(2x)$ $\quad y(3) = 9\sin(3)$

$\left.\frac{dy}{dx}\right|_{x=3} = 9\cos 3 + 6\sin(3)$

$\boxed{y-(9\sin(3)) = (9\cos 3 + 6\sin 3)(x-3)}$ T

$\boxed{y - 9\sin 3 = \frac{-1}{9\cos 3 + 6\sin 3}(x-3)}$ N

8. $\frac{dy}{dx} = -\sqrt{2}\sin x$ $\quad y(\pi/4) = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 1$

$\left.\frac{dy}{dx}\right|_{x=\pi/4} = -\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = -1$

$\boxed{y-1 = -(x-\pi/4)}$ T

$\boxed{y-1 = (x-\pi/4)}$ N

9. $\frac{dy}{dx} = -\csc^2 x + 2\csc x\cot x = 0$ $\quad \csc x(2\cot x - \csc x) = 0$

$\csc x = 0$ (never) $\quad 2\cot x - \csc x = 0$

$2\cot x = \csc x$

$2\frac{\cos x}{\sin x} = \frac{1}{\sin x}$

$2\cos x = 1$

$\cos x = \frac{1}{2}$ $\quad x = \frac{\pi}{3}$

There are more :)

$y\left(\frac{\pi}{3}\right) = 4 + \cot\left(\frac{\pi}{3}\right) - 2\csc\left(\frac{\pi}{3}\right) = 4 - \sqrt{3}$

$\boxed{y = 4 - \sqrt{3}}$

10. $\frac{dy}{dx} = -\csc x\cot x$

$\frac{d^2y}{dx^2} = -\csc x(-\csc^2 x) + \cot x(\csc x\cot x)$

Solutions

Additional Problems

1. Find $\dfrac{d^{87}(\sin x)}{dx^{87}}$ (the 87th derivative of sin *x*)

1: $\dfrac{d \sin x}{dx} = \cos x$

2: $\dfrac{d^2 \sin x}{dx^2} = -\sin x$

3: $\dfrac{d^3 \sin x}{dx^3} = -\cos x$

4: $\dfrac{d^4 \sin x}{dx^4} = \sin x$

$$\begin{array}{r} 21 \text{ R } 3 \\ 4\overline{)87} \\ \underline{84} \\ 3 \end{array}$$

$$\frac{d^{87}\sin x}{dx^{87}} = \boxed{-\cos x}$$

2. Let $f(x) = \cos x$. Find all positive integers *n* for which $f^{n}(x) = \sin x$.

$\dfrac{df}{dx} = -\sin x$

$\dfrac{d^2f}{dx^2} = -\cos x$

$\dfrac{d^2f}{dx^3} = \sin x$

$\dfrac{d^4f}{dx^4} = \cos x$

$$\begin{cases} n = 3 + 4k & \text{for } k = 0, 1, 2, \ldots \\ \text{or} & \\ n = 3 + 4(k-1) & \text{for } k = 1, 2, 3, \ldots \end{cases}$$

3. Find $\displaystyle\lim_{x\to 0}\frac{\tan(x+y)-\tan(y)}{x}$.

$$= \lim_{x\to 0}\frac{\tan(y+x)-\tan(y)}{x} \quad \text{think} \quad \lim_{h\to 0}\frac{\tan(y+h)-\tan(y)}{h}$$

$$= \frac{d(\tan y)}{dy} = \boxed{\sec^2 y}$$

4. Let $y = 3 + 2\sin x$.

(a) Find the x-coordinate of all points on the graph at which the tangent line is parallel to the line $y = \sqrt{2}x - 5$

(b) Find an equation of the tangent line to the graph at the point on the graph with *x*-coordinate $\pi/6$.

$\dfrac{dy}{dx} = 2\cos x$

$2\cos x = \sqrt{2}$

$\cos x = \dfrac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + 2\pi k \;\cup\; \overset{\frac{7\pi}{4}}{\cancel{\frac{3\pi}{4}}} + 2\pi k$

where k is an integer.

$y = f(x)$: $f(\frac{\pi}{6}) = 3 + 2\sin(\frac{\pi}{6}) = 3 + 2(\frac{1}{2}) = 4$

$f'(\frac{\pi}{6}) = 2\cos(\frac{\pi}{6}) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$$\boxed{y - 4 = \sqrt{3}\left(x - \frac{\pi}{6}\right)}$$