BC. Q101. LESSON 2 HW Solutions

① $f'(1.57) \approx \dfrac{f(1.74) - f(1.39)}{1.74 - 1.39} = \dfrac{1126 - 1255}{0.35}$

$= -368.57$

$f'(3) \approx \dfrac{f(3.24) - f(2.64)}{3.24 - 2.64} = \dfrac{805 - 869}{0.6}$

$= -106\frac{2}{3}$

② $f'(1) = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \,?$

▫ $f'_+(1) = \lim_{h\to 0^+} \dfrac{f(1+h) - f(1)}{h} = \lim_{h\to 0^+} \dfrac{\frac{1}{1+h} - [1]}{h} = \lim_{h\to 0^+} \dfrac{\frac{1}{1+h} - \frac{1+h}{1+h}}{h}$

$= \lim_{h\to 0^+} \dfrac{\frac{-h}{1+h}}{h} = \lim_{h\to 0^+} \dfrac{-1}{1+h} = -1$

▫ $f'_-(1) = \lim_{h\to 0^-} \dfrac{f(1+h) - f(1)}{h} = \lim_{h\to 0^-} \dfrac{1+h - [1]}{h}$

$= \lim_{h\to 0^-} 1 = 1$

$\therefore \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$ DNE ; $f'(1)$ DNE

f is not differentiable at $x = 1$

f has a corner at $x = 1$

③ a) i) $b(1) = -2+1 = -1$

ii) $\lim_{x\to 1^+} b(x) = \lim_{x\to 1^+} x = 1$

$\lim_{x\to 1^-} b(x) = \lim_{x\to 1^-} -2+x = -1$

$\therefore \lim_{x\to 1} b(x)$ DNE

iii) $\lim_{x\to 1} b(x) \neq b(1)$

$\therefore$ b is not continuous at $x=1$

b) $b'(1) = \lim_{h\to 0} \frac{b(1+h)-b(1)}{h} = ?$

$\square$ $b'_+(1) = \lim_{h\to 0^+} \frac{b(1+h)-b(1)}{h} = \lim_{h\to 0^+} \frac{(1+h)-(-2+1)}{h}$

$= \lim_{h\to 0^+} \frac{2+h}{h} = \lim_{h\to 0^+} \frac{2}{h} + \lim_{h\to 0^+} \frac{h}{h}$

$= \lim_{h\to 0^+} \frac{2}{h}^{\nearrow\infty} + \lim_{h\to 0^+} 1 \longrightarrow$ DNE

OPTIONAL: $\square$ $b'_-(1) = \lim_{h\to 0^-} \frac{b(1+h)-b(1)}{h} = \lim_{h\to 0^-} \frac{-2+(1+h)-(-2+1)}{h}$

$= \lim_{h\to 0^-} \frac{h}{h} = \lim_{h\to 0^-} 1 = 1$

$\therefore \lim_{h\to 0} \frac{b(1+h)-b(1)}{h}$ DNE ; $b'(1)$ DNE

The function b is not differentiable at $x=1$

(DISCONTINUOUS AT $x=1$)

c) If we first prove $b(x)$ is not continuous at $x=1$, then we simply need to state that $b(x)$ is not differentiable at $x=a$ by the THM we proved in class.

(4) $m'(1) = \lim_{h\to 0} \frac{m(1+h) - m(1)}{h} = ?$

$$m'_{+}(1) = \lim_{h\to 0^+} \frac{m(1+h) - m(1)}{h} = \lim_{h\to 0^+} \frac{\sqrt{1+h} - [1]}{h}$$

$$= \lim_{h\to 0^+} \frac{(\sqrt{1+h} - 1)}{(h)} \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$$

$$= \lim_{h\to 0^+} \frac{1 + h - 1}{h(\sqrt{1+h} + 1)} = \lim_{h\to 0^+} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{2}$$

$$m'_{-}(1) = \lim_{h\to 0^-} \frac{m(1+h) - m(1)}{h} = \lim_{h\to 0^-} \frac{\frac{1+h}{2} + \frac{1}{2} - [1]}{h}$$

$$= \lim_{h\to 0^-} \frac{\frac{1}{2} + \frac{h}{2} + \frac{1}{2} - 1}{h} = \lim_{h\to 0^-} \frac{\frac{h}{2}}{h}$$

$$= \lim_{h\to 0^-} \frac{1}{2} = \frac{1}{2}$$

$\therefore \lim_{h\to 0} \frac{m(1+h) - m(1)}{h} = \frac{1}{2} \quad m'(1) = \frac{1}{2}$

m is differentiable at $x = 1$

m is <u>smooth</u> at $x = 1$

⑤ For g to be continuous at $x=1$ we know that the limit must exist.

In order for the limit to exist...

$$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^-} g(x)$$

$$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} ax^2+bx = a+b$$

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} 3-x = 2$$

$$\therefore \quad a+b=2$$

Now for g to be differentiable at $x=1$

must be equal

$$g'_+(1) = \lim_{h\to 0^+} \frac{g(1+h)-g(1)}{h} = \lim_{h\to 0^+} \frac{a(1+h)^2+b(1+h) - [2]}{h}$$

$$= \cdots = 2a+b$$

$$g'_-(1) = \lim_{h\to 0^-} \frac{g(1+h)-g(1)}{h} = \lim_{h\to 0^-} \frac{3-(1+h) - [2]}{h}$$

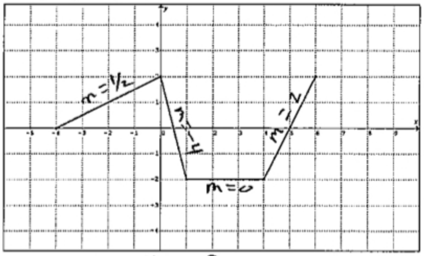
$$= \cdots = -1$$

so $2a+b=-1$

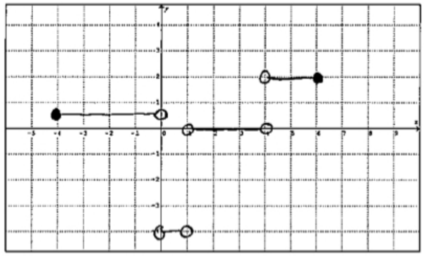
$$\begin{cases} a+b=2 \\ 2a+b=-1 \end{cases} \quad \therefore \quad a=-3,\ b=5$$

[redacted]

6 The graph of the function $y = f(x)$ shown here is made of line segments joined end to end. Graph $y = f'(x)$ and state its domain.



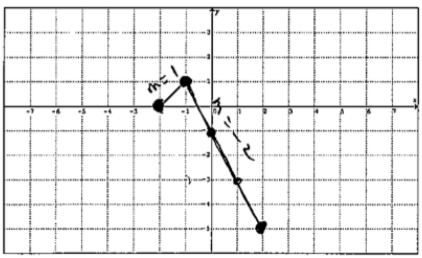
$y = f(x)$



$y = f'(x)$

7 Sketch the graph of a continuous function with domain [-2,2], $f(0) = -1$, and

$$f'(x) = \begin{cases} 1; & x < -1 \\ -2; & x > -1 \end{cases}.$$



$y = f(x)$

8 Using the information from problem 2, write an equation of the line tangent to f at $x = 0$.

$f(0) = -1$

$f'(0) = -2$

$y + 1 = -2(x)$

9 [redacted]
$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{5(x+h)^2-2(x+h)+1-[5x^2-2x+1]}{h}$$

$$= \lim_{h\to0} \frac{\cancel{5x^2}+10xh+5h^2-\cancel{2x}-2h\cancel{+1}-\cancel{5x^2}+\cancel{2x}-\cancel{1}}{h}$$

$$= \lim_{h\to0} \frac{h(10x+5h-2)}{h} = \lim_{h\to0} 10x+5h-2 = 10x-2$$

$$\boxed{f'(x) = 10x-2}$$

10 [redacted]
$$\boxed{\begin{aligned} f'(x) &= 0 \\ 10x-2 &= 0 \\ &\boxed{x = 1/5} \end{aligned}}$$

11 [redacted]
$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h\to0} \frac{\frac{x+1-(x+h+1)}{(x+1)(x+h+1)}}{h}$$

$$= \lim_{h\to0} \frac{\frac{-h}{(x+1)(x+h+1)}}{h} = \lim_{h\to0} \frac{-1}{(x+1)(x+h+1)} = \frac{-1}{(x+1)^2}$$

12 [redacted]
$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}$$

$$= \lim_{h\to0} \frac{\left(\sqrt{2(x+h)+1}-\sqrt{2x+1}\right)}{h} \frac{\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}{\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$

$$= \lim_{h\to0} \frac{2(x+h)+1-(2x+1)}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)} = \lim_{h\to0} \frac{2h}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$

$$= \lim_{h\to0} \frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

(13) f diff at $x=1$

$\therefore \lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$ exists

Now $f'(1) = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h \to 0} \frac{f(1+h)}{h} - \lim_{h \to 0} \frac{f(1)}{h}$

$= [5] - \lim_{h \to 0} \frac{0}{h}$

$= 5 - \lim_{h \to 0} 0 = 5 - 0 = \boxed{5}$

(14) $f(0) = f(0+0) = f(0) + f(0) + 5(0)(0)$

$\therefore f(0) = 2f(0)$

subtraction

$0 = f(0)$

Now: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{f(x)+f(h)+5xh-f(x)}{h}$

$= \lim_{h \to 0} \frac{f(h)+5xh}{h} = \lim_{h \to 0} \frac{f(h)}{h} + \lim_{h \to 0} 5x$

$= 3 + 5x$

$\boxed{f'(x) = 3 + 5x}$

$\lim_{h \to 0} 5x$

h varies, so "5x" is a constant in relationship to h.