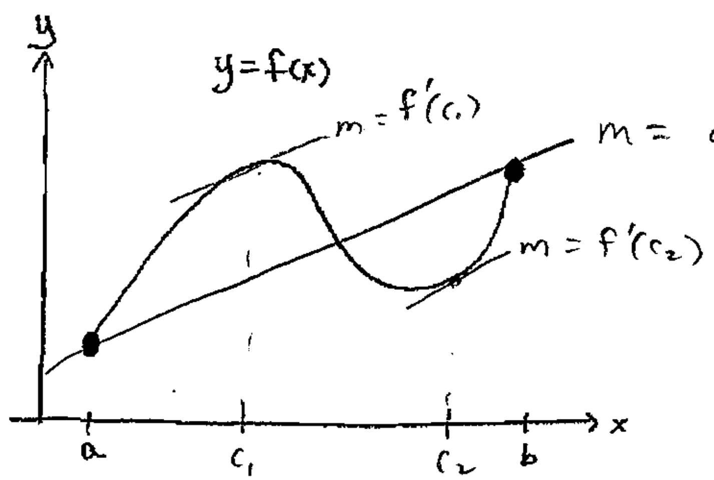
Q204.LESSON3.MEAN VALUE THEOREM (VERSION 2.0)

MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exist at least one point $x = c$ in (a, b) such that:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

"instant Δ = average Δ"

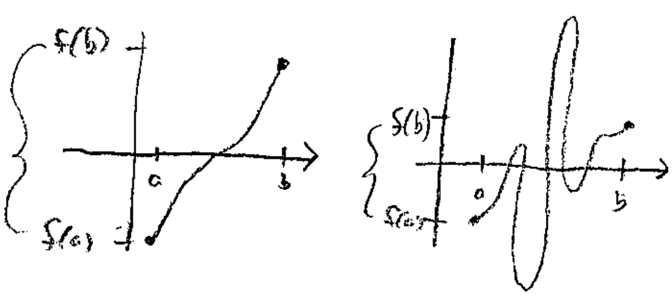


$m =$ ave rate Δ $= \dfrac{f(b)-f(a)}{b-a}$

INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$.

If f' is continuous on the closed interval $[a, b]$, then f' takes on every value between $f'(a)$ and $f'(b)$.



MVT: TECHNOLOGY REQUIRED

EX1. Consider the function: $f(x) = x\cos(x^2)$ on [1.5, 3]

f is cont. on $[1.5, 3]$ f is diff on $(1.5, 3)$

Find the value(s) of x on $1.5 < x < 3$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

$$\underset{[1.5,\,3]}{\text{Ave rate } \Delta} = \frac{f(3) - f(1.5)}{3 - 1.5} = -1.194$$

CALCULATOR : (y1(3) - y1(1.5)) / (3 - 1.5)

$$f'(x) = -1.194 \quad \text{at} \quad \left.\begin{array}{l} x = 1.764 \\ x = 2.540 \end{array}\right\} \text{on } (1.5, 3)$$

CALCULATOR :

HOME: d(y1(x), x)

(COPY ANSWER)

y2 = (Paste Answer)

<u>OPTION 1</u>

$y2 = f'(x)$

$y3 = -1.194$

(FIND INTERSECTION)

<u>OPTION 2</u>

$y2 = f'(x) + 1.194$

(FIND ZERO)

$\rightarrow f'(x) = -1.194$

so

$f'(x) + 1.194 = 0$

2
16
4
64

-8+1

MVT: NO TECHNOLOGY PERMITTED

EX2. Consider the function $g(x) = x^3 + 1$ on $-2 \le x \le 4$.

g is cont. on [-2,4] g is diff on (-2,4)

Find the value(s) of x on $-2 < x < 4$ that satisfy the conclusion to the Mean Value Theorem.

$$\underset{[-2,4]}{\text{Ave rate } \Delta} = \frac{g(4) - g(-2)}{4-(-2)} = \frac{\left((4)^3+1\right) - \left((-2)^3+1\right)}{6}$$

$$= \frac{65 + 7}{6} = 12$$

$$g'(x) = \overbrace{3x^2}^{\text{instant}} = \overbrace{12}^{\text{ave}}$$

$$x^2 = 4$$

$$x = -2 \text{ or } \boxed{x = 2} \text{ on } (-2,4)$$

not on (-2,4)

answer

MVT: TECHNOLOGY REQUIRED

PRACTICE1: Consider the function: $f(x) = x^{-2x} + x^3$ on [1.8, 3.3]

f is cont on $[1.8, 3]$ f is diff on $(1.8, 3.3)$

Find the value(s) of x on $1.8 < x < 3.3$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

$$\underset{[1.8,\,3.3]}{\text{Ave rate } \Delta} = \frac{f(3.3) - f(1.8)}{3.3 - 1.8} = 19.990$$

$$f'(x) = 19.990 \quad \text{at} \quad \boxed{x = 2.583} \quad \text{on } (1.8, 3.3)$$

MVT: NO TECHNOLOGY PERMITTED

PRACTICE2: Consider the function $g(x) = 4 + \sqrt{x-1}$ on $1 \le x \le 5$.

g is cont. on [1, 5] g is diff on (1, 5)

Find the value(s) of x on $1 < x < 5$ that satisfy the conclusion to the Mean Value Theorem.

$$\text{Ave rate } \Delta \; [1,5] = \frac{g(5)-g(1)}{5-1} = \frac{(4+\sqrt{4})-(4+\sqrt{0})}{5-1} = \frac{2}{4} = \frac{1}{2}$$

$$g'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}} = \frac{1}{2}$$

$$\frac{1}{\sqrt{x-1}} = 1 \longrightarrow \frac{\sqrt{x-1}}{1} = 1$$

$$x - 1 = 1$$

$$\boxed{x = 2} \text{ on } (1, 5)$$

* NOTE g is not diff at x = 1 but that is not required

NOTES: MVT + IVT

1. Consider a function $y = f(x)$ which is both continuous and differentiable on $3 \le x \le 8$

Select values of x and $f(x)$ are shown in the table below.

x	3	4	8
$f(x)$	6	12	3

Prove that $f'(x) = 0$ at least one time on $3 \le x \le 8$.

✓ f is continuous on [3, 8]
✓ f is differentiable on [3, 8]
∴ f′ is continuous on [3, 8]

Ave rate Δ on [3, 4] $= \frac{12-6}{4-3} = 6$

∴ $f'(x) = 6$ at least one time on (3, 4) M.V.T

Ave rate Δ on [4, 8] $= \frac{3-12}{8-4} = \frac{-9}{4}$

∴ $f'(x) = -\frac{9}{4}$ at least one time on (4, 8) M.V.T

f′ goes from 6 to $-\frac{9}{4}$ because f′ is continuous.
∴ $f'(x) = 0$ at least one time on [3, 8]
I.V.T.

2. Consider a function $y = f(x)$ which is both continuous and differentiable on $2 \le x \le 5$

Select values of x and $f(x)$ are shown in the table below.

x	2	4	5
$f(x)$	3	16	16

Prove that $f'(x) = 0$ at least one time on $2 \le x \le 5$.

✓ f is continuous on [2, 5]
✓ f is differentiable on (2, 5)

Ave rate Δ on [4, 5] $= \frac{16-16}{5-4} = 0$

∴ $f'(x) = 0$ at least one time on (4, 5) M.V.T (OR ROLLE'S THM)

$f'(x) = 0$ at least on time on [2, 5]