Q204 VIDEO L3 EXAMPLES 1–7

Q204.LESSON2.L'HOPITAL'S RULE (VERSION 2.0)

$\lim \to \frac{0}{\#} = 0$

$\lim \to \frac{\#}{0} \to \begin{matrix}\infty \\ (DNE) \\ -\infty\end{matrix}$

$\lim \to \frac{\#}{\infty} = 0$

$\lim \to \frac{\infty}{\#} \to \infty$

$\lim \to \frac{0}{0}$ INDETERMINANT

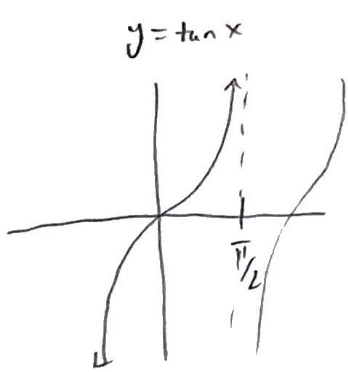
$\lim \to \frac{\infty}{\infty}$ INDETERMINANT

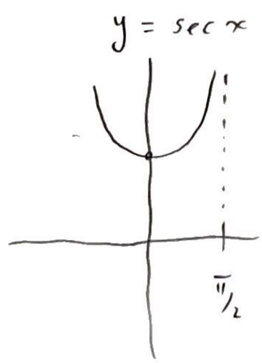
$\lim \to 0 \cdot \# = 0$

$\lim \to \infty \cdot \# \to \infty$

$\lim \to 0 \cdot \infty$ INDETERMINANT

INTRO





① $\lim_{x\to 0} \frac{\cos x + 2x - 1}{3x} \quad \left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{-\sin x + 2}{3} = \boxed{\frac{2}{3}}$

② $\lim_{x\to 0} \frac{e^{x} + e^{-x} - 2}{1 - \cos(2x)} \quad \left\{\frac{0}{0}\right\}$

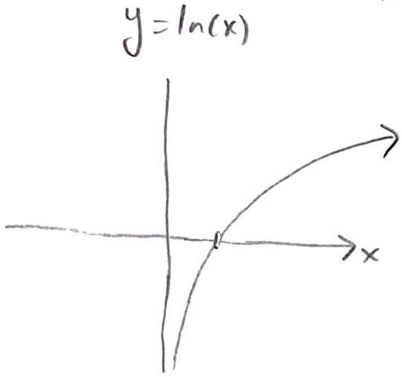
$= \lim_{x\to 0} \frac{e^{x} - e^{-x}}{-2\sin(2x)} \quad \left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{e^{x} + e^{-x}}{-4\cos(2x)} = \frac{2}{-4} = \boxed{-\frac{1}{2}}$

③ $\lim_{x\to \frac{\pi}{2}^-} \frac{4\tan x}{1 + \sec x} \quad \left\{\frac{\infty}{\infty}\right\}$ $\quad \frac{1}{\cos}$

$= \lim_{x\to \frac{\pi}{2}^-} \frac{4\sec^2 x}{\sec x \tan x} \underset{\text{simplify}}{=} \lim_{x\to \pi/2^-} \frac{4\sec x}{\tan x}$

$\underset{\text{simplify}}{=} \lim_{x\to \frac{\pi}{2}^-} \frac{4}{\sin x} = \frac{4}{1} = \boxed{4}$



4. $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}} \quad \left\{\dfrac{\infty}{\infty}\right\}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$

$= \lim_{x\to\infty} \dfrac{1}{x} \, 2\sqrt{x}$

$= \lim_{x\to\infty} \dfrac{2}{\sqrt{x}} = \boxed{0}$

5. $\lim_{x\to\infty} \dfrac{e^{3x}}{x^2} = \lim_{x\to\infty} \dfrac{3e^{3x}}{2x} = \lim_{x\to\infty} \dfrac{9e^{3x}}{2} \to \boxed{\infty}$

6. $\lim_{x\to 0^+} x^2 \cdot \ln(x) \quad -\{0\cdot\infty\}$

$= \lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x^2}} \quad \left\{\dfrac{\infty}{\infty}\right\}$

$= \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \dfrac{x^2}{-2} = \boxed{0}$

7. $\lim_{x\to \frac{\pi}{2}^-} (2x-\pi)\sec x \quad \{0\cdot\infty\}$

$= \lim_{x\to \frac{\pi}{2}^-} \dfrac{2x-\pi}{\cos x} \quad \left\{\dfrac{0}{0}\right\}$

$= \lim_{x\to \pi/2^-} \dfrac{2}{-\sin x} = \dfrac{2}{-1} = \boxed{-2}$

LINEARIZATION (SAME AS A TANGENT LINE TO A CURVE)

EX1. A. Find a linearization to $f(x)=3x^3-2x^6+1$ at $x=1$.

tangent line

B. Use the linearization to approximate ~~$f(0.3)$~~ $f(1.03)$

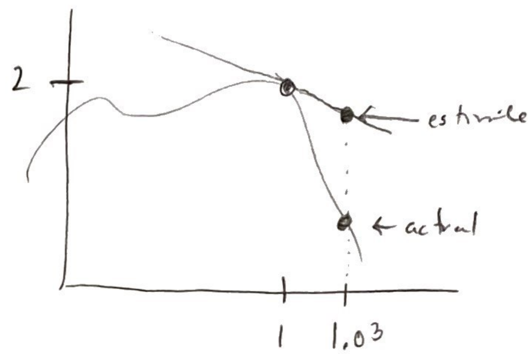
A] $L(x) = f(1) + f'(1)(x-1)$

$f(1) = 3-2+1 = 2$

$f'(x) = 9x^2 - 12x^5$

$f'(1) = 9-12 = -3$

$\boxed{L(x) = 2 - 3(x-1)}$



B] $f(1.03) \approx L(1.03) = 2 - 3(1.03-1) = 2-3(0.03)$

$= 2 - 0.09$

$= 1.91$

$= \boxed{1.91}$

EX2. A. Find a linearization to $f(x)=\cos^2(x)+2x$ at $x=0$.

B. Use the linearization to approximate $f(0.2)$

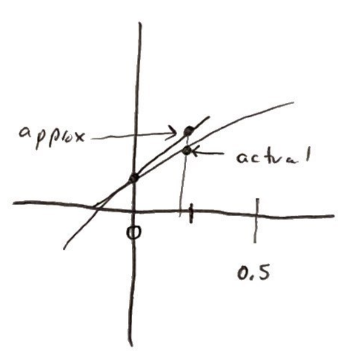
A] $L(x) = f(0) + f'(0)(x-0)$

$f(0) = \cos^2(0) + 2(0) = 1$

$f'(x) = 2\cos(x)(-\sin x) + 2$

$f'(0) = -2(1)(0) + 2 = 2$

$\boxed{L(x) = 1 + 2(x-0)}$



B] $f(0.2) \approx L(0.2) = 1 + 2(0.2-0) = 1+2(0.2) = \boxed{1.4}$

EX3. Consider the function $y = f(x)$

with $f(2) = 5$ and derivative $f'(x) = e^{(x-2)} + \sqrt{\cos(x-2) + 3x/2}$

A. Find a linearization to $y = f(x)$ at $x = 2$

B. Use the linearization to approximate $f(2.1)$

A] $L(x) = f(2) + f'(2)(x-2)$

$f(2) = 5$ given

$f'(x) =$ given

$f'(2) = e^0 + \sqrt{\cos(0) + 3} = 1 + \sqrt{1+3} = 1 + 2 = 3$

$L(x) = 5 + 3(x-2)$

B] $f(2.1) \approx L(2.1) = 5 + 3(2.1 - 2)$

$= 5 + 3(0.1)$

$= \boxed{5.3}$