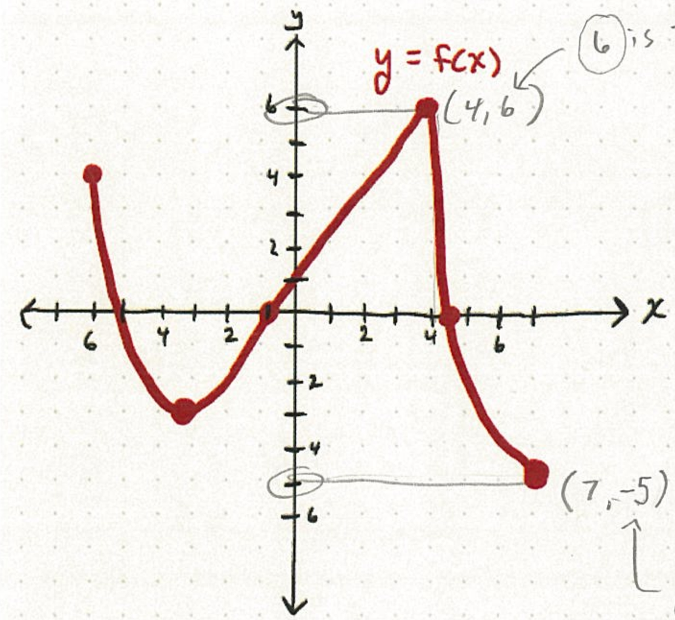
MASTER COPY

Q204.LESSON1.ABSOLUTE EXTREMES (VERSION 2.0)

(6) IS THE ABS. MAX



where is absolute max?

x = 4

where is absolute min?

x = 7

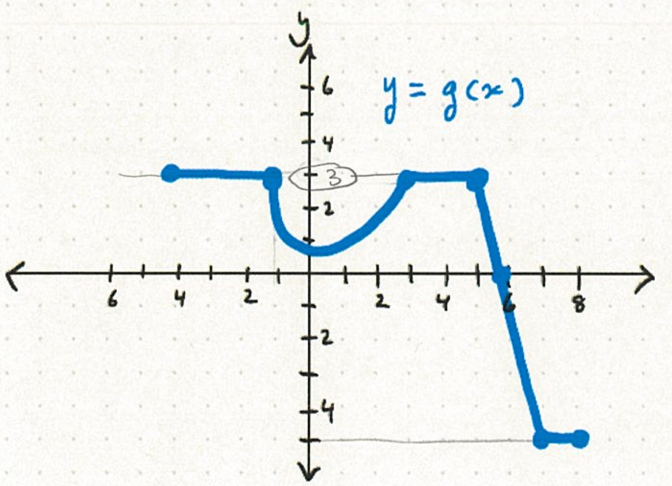
what is the absolute max?

f = 6

what is the absolute min?

f = -5

(-5) IS THE ABS. MIN



where is absolute max?

x ∈ [4, 1] ∪ [3, 5]

where is absolute min?

x ∈ [7, 8]

what is the absolute max?

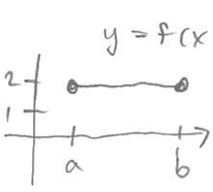
f = 3

what is the absolute min?

f = -5

EXTREME VALUE THEOREM

If a function is continuous on a closed interval [*a*, *b*], then the function is guaranteed to have an absolute maximum and an absolute minimum value.



abs. max = 2

abs. min = 2

CLOSED INTERVAL TEST

Consider a function that is continuous on a closed interval $[a, b]$.

Find all the critical x-values on the open interval (a, b).

Evaluate the function at each of the interior critical x-values and each of the endpoints.

The absolute maximum is the largest and the absolute minimum is the smallest of these function's values.

EXAMPLE: (NO TECHNOLOGY) Use the closed interval test to find the absolute maximum and the absolute minimum value of $f(x) = \ln\left(x^2 + x + 1\right)$ on the closed interval $-1 \le x \le 1$.

Interior critical x values

$$f'(x) = \frac{1}{x^2+x+1} \cdot (2x+1) = \frac{2x+1}{x^2+x+1} = 0 \qquad x = -\frac{1}{2}$$

never zero here.

endpoint critical x-values

$x = -1$, $x = 1$

evaluation

$f(-1) = \ln(1) = 0$ ← zero

$f(-1/2) = \ln\left(\frac{1}{4} - \frac{1}{2} + 1\right) = \ln(3/4)$ ← negative

$f(1) = \ln(3)$ ← positive

Answer

absolute max $= \ln(3)$

absolute min $= \ln(3/4)$

ONE CRITICAL ON AN OPEN INTERVAL THEOREM

Suppose a continuous function f has exactly one critical x-value on the open interval (a, b).

If the function has a local extreme at the critical point, then the local extreme is also the absolute extreme value.

EXAMPLE: (NO TECHNOLOGY) Provided any exists, find the absolute extreme values of $f(x) = 2\pi x^2 + \dfrac{2000}{x}$ on the open interval $0 < x < 10$.

$$f'(x) = 4\pi x - \frac{2000}{x^2} = \frac{4\pi x^3 - 2000}{x^2} = 0$$

x^2 never zero when $0 < x < 10$

$$4\pi x^3 - 2000 = 0 \qquad x^3 = \frac{500}{\pi} \qquad x = \sqrt[3]{\frac{500}{\pi}}$$

This is between $(0, 10)$

$$f''(x) = 4\pi + \frac{4000}{x^3}$$

$f''\left(\sqrt[3]{\frac{500}{\pi}}\right) > 0 \quad \therefore \quad f$ has a local min at $x = \sqrt[3]{\frac{500}{\pi}}$ (2nd derivative Test)

Since $x = \sqrt[3]{\frac{500}{\pi}}$ is the only critical point on open $0 < x < 10$

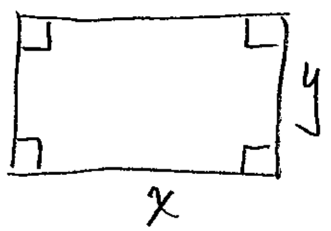
The absolute min also occurs at $x = \sqrt[3]{\frac{500}{\pi}}$

Answer

$$f\left(\sqrt[3]{\frac{500}{\pi}}\right) = \boxed{2\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2 + \frac{2000}{\sqrt[3]{\frac{500}{\pi}}}}$$ abs. min

OPTIMIZATION APPLICATION

EXAMPLE: (NO TECHNOLOGY) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence? Use calculus to justify your answer.



MAX Area = (base)(height)

$A = x \cdot y$

$P = 2x + 2y$

$100 = 2x + 2y$

$y = 50 - x$

Function: $A(x) = x(50 - x) = 50x - x^2$

Domain $0 < x < 50$

$A'(x) = 50 - 2x = 0$

$x = 25$

$A''(x) = -2$

$A''(25) = -2 < 0$

$\therefore$ A is local max at $x = 25$

$\therefore$ open interval so A is abs. max at $x = 25$

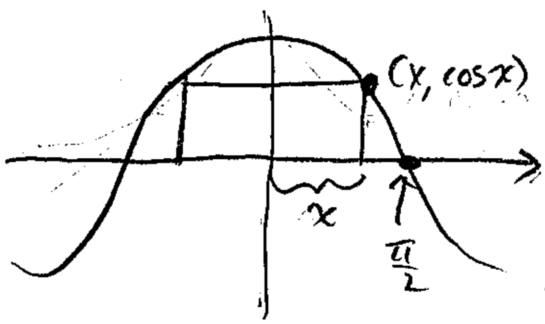
Answer: $A(25) = \underline{\underline{625 \text{ ft}^2}}$

absolute max area

OPTIMIZATION APPLICATION

EXAMPLE: (TECHNOLOGY REQUIRED) A rectangle is to be inscribed under one arch of the cosine curve. What is the largest area the rectangle can have?

Curve is symmetric about y-axis



max Area = (base)(height)

$A(x) = (2x)(\cos x)$

Domain $0 < x < \pi/2$

TECH; $A'(x) = 0$ at $x = \underline{0.860}$

First Derivative Test

A has local max at $x = \underline{0.860}$
b/c $A'(x)$ goes from positive to negative
at $x = \underline{0.860}$

Answer: $A(0.860) = \underline{1.122}$ units^2

absolute max

"x is distance from origin to right end of rectangle"

HOMEWORK

CLOSED INTERVAL TEST (NO TECHNOLOGY)

A. Use the closed interval test to find the absolute maximum and the absolute minimum value of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the closed interval $-2 \le x \le 3$.

ONE CRITICAL ON OPEN THEOREM (NO TECHNOLOGY)

B. Provided it exists, find the absolute extreme value of $f(x) = 2 + \ln(\sin x + 2)$ on the open interval $1 < x < 2$. Use calculus to justify your answer.

ONE CRITICAL ON OPEN THEOREM (TECHNOLOGY REQUIRED)

C. Provided it exists, find the absolute extreme value of $f(x) = x^2 \sin(x + 2.1)$ on the open interval $4 < x < 7$. Use calculus to justify your answer.

OPTIMIZATION APPLICATION (NO TECHNOLOGY)

1. What is the smallest perimeter possible for a rectangle whose area is 16 cm^2, and what are its dimensions? Use calculus to justify your answer.

2. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions? Use calculus to justify your answer.

OPTIMIZATION APPLICATION (TECHNOLOGY REQUIRED)

3. A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -\pi$ to $x = \pi$. What is the largest area of this rectangle can have? Use calculus to justify your answer.

4. A rectangle has its base on the x-axis and is inscribed under the curve $y = \dfrac{4.6 - x^2}{1 + x^2}$.
What is the largest area of this rectangle can have? Use calculus to justify your answer.

RELATED RATES REVIEW (NO TECHNOLOGY)

5. Water drains from the conical tank shown in the figure at a rate of 5 ft^3/min.

How fast is the water level changing when $h = 6$ ft?