AB. Q100. LESSON 2. GRAPH RED 3

A1. D: $\{x \mid -3 \leq x \leq 24\}$ or $x \in [-3, 24]$

* some approximations

A2. R: $\{y \mid 2.75 \leq y \leq 13.5\}$ or $y \in [2.75, 13.5\}$

B1. f is continuous on $[-3, 4) \cup (4, 21) \cup (21, 7]$

B2. f is not continuous at $x = 4$ and $x = 21$

at $x = 4$

i) $f(4) = 8$

ii) $\lim_{x \to 4^-} f(x) = 11$

$\lim_{x \to 4^+} f(x) = 8$

$\therefore \lim_{x \to 4} f(x)$ DNE

iii) $\lim_{x \to 4} f(x) \neq f(4)$

at $x = 21$

i) $f(21) = 7$

ii) $\lim_{x \to 21} f(x) = 5$

iii) $\lim_{x \to 21} f(x) \neq f(21)$

* Concept version uses all paretheses

C1.* f is increasing on $[-3, 4) \cup [6.5, 12] \cup [19.5, 21) \cup (21, 24]$
b/c the slope of f is positive on $(-3, 4) \cup (6.5, 12) \cup (19.5, 21) \cup (21, 24)$

Technically speaking $\left.\begin{array}{l} f'(-3) \text{ is positive} \\ f'(24) \text{ is positive} \end{array}\right.$

C2.* f is decreasing on $(4, 6.5] \cup [12, 13] \cup [13, 19.5]$
b/c the slope of f is negative on $(4, 6.5) \cup (12, 13) \cup (13, 19.5)$

D1. f has a relative max at $x = 12$ because at this x-value f is continuous and the slope of f goes from positive to negative.

- f has a relative endpoint max at $x = 24$
- f has a relative max at $x = 21$
  f at $x = 21$ is relatively larger than f at value around $x = 21$

* AT $x = 4$ f has neither a local max nor a local min

D2. f has a relative min at $x = 6.5, 19.5$ because at these x-values f is continuous and the slope of f goes from negative to positive.

- f has a relative endpoint min at $x = -3$

E1. f is concave up on $(4, 8.5) \cup (18, 20.5)$ because the slope of f is increasing on this interval

E2. f is concave down on $(-3, 4) \cup (8.5, 13) \cup (17, 18) \cup (20.5, 21) \cup (21, 24)$ because the slope of f is decreasing on this interval.

* There is no concavity at $x = 21$

F1. Ave slope over $[4, 17] = \dfrac{f(17) - f(4)}{17 - 4} = \dfrac{7 - 8}{13} = \dfrac{-1}{13}$

F2 Ave $f'(x)$ over $[13, 17] = \dfrac{11 - 7}{13 - 17} = \dfrac{4}{-4} = -1$

G1. Slope at $x = 15$: $m = -1$ $\frac{\Delta y}{\Delta x}$

G2. $f'(-1.5) = \dfrac{f(0) - f(-3)}{0 - (-3)} = \dfrac{10 - 7}{3} = \dfrac{3}{3} = 1$ average small neighborhood

G3. $f'(17) = -1$ smooth slope transition

G4. $f'(19.5) = 0$ eyeball horizontal tangent

G5. $f'(13)$ DNE SHARP TRANSITION

G6. $f'(4)$ DNE f is not continuous at $x = 4$ so f cannot have a slope at $x = 4$

G7. $f'(21)$ DNE f is not continuous at $x = 21$, so f cannot have a slope at $x = 21$.

H. f has a horizontal tangent at approximately

$x = 6.5,\ 12,\ 19.5$

I. f does not have a slope at $x = 4, 13, 21$

at $x=4$

$\lim_{x \to 4} f(x) \neq f(4)$

NOT cont at $x=4 \rightarrow$ NOT slope at $x=4$

at $x=13$

$f'_{-}(13) \neq f'_{+}(13)$

at $x=21$

$\lim_{x \to 21} f(x) \neq f(21)$

NOT CONT. at $x=21 \rightarrow$ NO slope at $x=21$

J. MVT APPLIES ON $[4,13]$ because

f is cont. on $[4,13]$

f has a slope on $(4,13)$

$$\text{Ave rate }\Delta_{[4,13]} = \frac{f(13)-f(4)}{13-4} = \frac{11-8}{13-4} = \frac{1}{3}$$

at approx $x = 7$ and $x = 10$

$f'(7) = \frac{1}{3} \quad f'(10) = \frac{1}{3}$

K. $y - 11 = -(x - 13)$

L. $f'(20) = 1.12$ Given

$y - 3 = 1.12(x - 20)$