LESSON 2 CLASS PRACTICE SET — SOLUTIONS

(1.) $\frac{dy}{dx} = y^3 + 2x$ w/ $y = f(x)$. $f(1) = 2 \longrightarrow (x=1, y=2)$

$f'(x,y) = y^3 + 2x \quad f'(1) = f'(1,2) = (2)^3 + 2(1) = \boxed{10}$

$f'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(y^3 + 2x\right) = 3y^2\frac{dy}{dx} + 2$

$f''(1) = f''\left(x=1, y=2, \frac{dy}{dx} = 10\right) = 3(2)^2(10) + 2 = \boxed{122}$

(2.) $\frac{dy}{dx} = \frac{3x^2+1}{2y} \longrightarrow f'(x,y) = \frac{3x^2+1}{2y} \quad f(1) = 4 \rightarrow (x=1, y=4)$

a) $f'(1,4) = \frac{3(1)^2+1}{2(4)} = \boxed{\frac{1}{2}}$

b) $L(x) = f(1) + f'(1,4)(x-1)$

$L(x) = 4 + \frac{1}{2}(x-1)$

$f(1.2) \approx L(1.2) = 4 + \frac{1}{2}(0.2) = \boxed{4.1}$

c) $f'(1) = f'(1,4) = \frac{1}{2}$

quotient Rule

$f'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{3x^2+1}{2y}\right) = \frac{2y(6x) - (3x^2+1)(2)\frac{dy}{dx}}{(2y)^2}$

$f''(1) = f''\left(x=1, y=2, \frac{dy}{dx} = \frac{1}{2}\right) = \frac{2(4)(6)(1) - (3+1)(2)\left(\frac{1}{2}\right)}{(8)^2} = \frac{44}{64} = \frac{11}{16} > 0$

The graph of f is concave up at the point $(1,4)$. Linear approximations, centered at $(1,4)$, will underestimate f for x values near $x=1$. ∴ The approximation in part (b) is an under-estimation.

D) $\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$y = \pm\sqrt{x^3+x+C}$

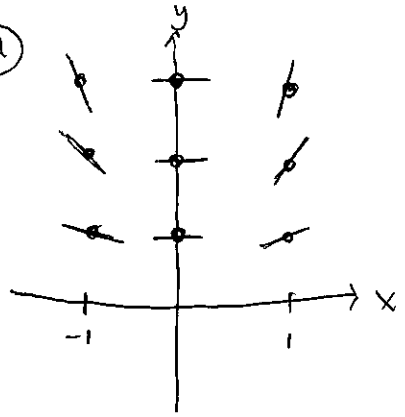
$4 = +\sqrt{1+1+C}$

$\therefore C = 14$

$\boxed{y = \sqrt{x^3+x+14}}$

(3) $\frac{dy}{dx} = \frac{xy}{2}$

(a)



(b) $f(0) = 3 \rightarrow (x=0, y=3)$ $\quad f'(0,3) = 0$

$L(x) = f(0) + f'(0,3)(x-0)$

$L(x) = 3 + 0(x-0)$

$\boxed{L(x) = 3}$ ← equation

$f(0.2) \approx L(0.2) = \boxed{3}$ ← answer

(c) ← product Rule

$$f'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{xy}{2}\right) = \frac{1}{2}\left(x\frac{dy}{dx} + y\right)$$

$$f''(0) = f''\left(x=0,\ y=3,\ \frac{dy}{dx}=0\right) = \frac{1}{2}(0\cdot 0 + 3) = \frac{3}{2} > 0$$

The approximation found in part (b) is an underestimation to $f(0.2)$

(D) $\int \frac{dy}{y} = \int \frac{x}{2}\,dx$

$\ln|y| = \frac{x^2}{4} + C$

$|y| = Ce^{x^2/4}$

$y = C^*e^{x^2/4}$

$3 = C^*e^{0}$ $\quad \therefore C^* = 3$

$\boxed{y = 3e^{x^2/4}}$