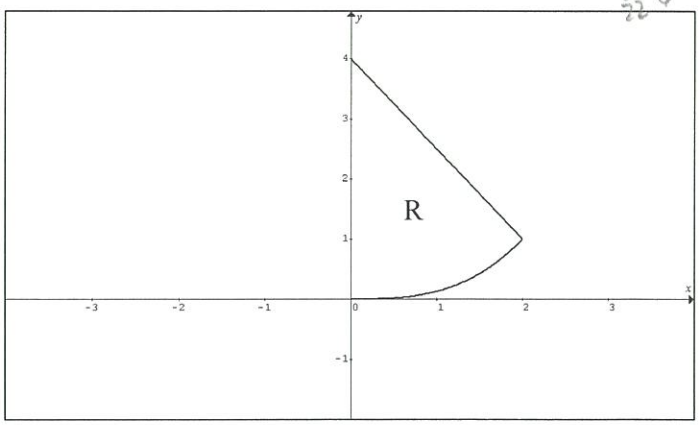
KEY

$x^3 = 8y$ $\quad y - 4 = -\frac{3}{2}x$

$x = 2y^{1/3}$ $\quad x = -\frac{2}{3}y + \frac{8}{3}$

Consider the region R, in the first quadrant, bounded by the graphs $y = \frac{x^3}{8}$ and $y = 4 - \frac{3}{2}x$ as shown in the figure.



a) Find the volume of the solid if R is revolved about the x-axis

$$V = \pi \int_0^2 \left[\left(4 - \tfrac{3}{2}x\right)^2 - \left(\tfrac{x^3}{8}\right)^2\right] dx$$

Anti-Derivative

$$= \pi \int_0^2 \left(16 - 12x + \tfrac{9}{4}x^2 - \tfrac{x^6}{64}\right) dx = \pi \left[16x - 6x^2 + \tfrac{3x^3}{4} - \tfrac{x^7}{7 \cdot 64}\right]_0^2$$

$$= \pi \left[16(2) - 6(4) + 3(2) - \frac{(2)^7}{7 \cdot 64}\right] = \pi \left[32 - 18 - \tfrac{2}{7}\right]$$

$$= \pi \left[\tfrac{224}{7} - \tfrac{126}{7} - \tfrac{2}{7}\right] = \boxed{\tfrac{96}{7}\pi}$$

b) Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid if R is revolved about the line $y = 8$.

$$V = \pi \int_0^2 \left[\left(8 - \tfrac{x^3}{8}\right)^2 - \left(8 - \left(4 - \tfrac{3}{2}x\right)\right)^2\right] dx$$

c) Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid if R is revolved about the line $y = -3$.

$$V = \pi \int_0^2 \left[\left(4 - \tfrac{3}{2}x + 3\right)^2 - \left(\tfrac{x^3}{8} + 3\right)^2\right] dx$$

d) Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid if R is revolved about the y-axis

$$V = \pi \int_0^1 \left(2y^{1/3}\right)^2 dy + \pi \int_1^4 \left(\tfrac{8}{3} - \tfrac{2}{3}y\right)^2 dy$$

e) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is a rectangle with height of 7. Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid.

$$A = 7\left(4 - \tfrac{3}{2}x - \tfrac{x^3}{8}\right) \qquad V = 7\int_0^2 \left(4 - \tfrac{3}{2}x - \tfrac{x^3}{8}\right) dx$$

f) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is a semi-circle. Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid.

$$A = \tfrac{1}{2}\pi \left(\frac{4 - \frac{3}{2}x - \frac{x^3}{8}}{2}\right)^2 = \tfrac{\pi}{8}\left(4 - \tfrac{3}{2}x - \tfrac{x^3}{8}\right)^2 \qquad V = \tfrac{\pi}{8}\int_0^2 \left(4 - \tfrac{3}{2}x - \tfrac{x^3}{8}\right)^2 dx$$

g) The base of a solid is the region R. Each cross section of the solid perpendicular to the y-axis is a square. Set up, but do not evaluate an expression involving one or more integral used to find the volume of the solid.

$$V = \int_0^1 \left(2y^{1/3}\right)^2 dy + \int_1^4 \left(\tfrac{8}{3} - \tfrac{2}{3}y\right)^2 dy$$

h) Set up, but do not evaluate an expression involving one or more integral used to find the perimeter of the region R.

$$P = 4 + \int_0^2 \sqrt{1 + \left(-\tfrac{3}{2}\right)^2}\, dx + \int_0^2 \sqrt{1 + \left(\tfrac{3}{8}x^2\right)^2}\, dx$$

$\frac{dy}{dx}$ $\qquad$ $\frac{dy}{dx}$

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