AB
Advanced Calculus

AB.Q403.REVIEW ASSESSMENT (PART F)

Derivative Tests and Miscellaneous Concepts

points

NO CALCULATORS

NAME: KEY

DATE:

BLOCK:

I *(print name)*______________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

$f'(x) \rightarrow$ increase / decrease

1. $f''(x) \rightarrow$ Concave up / Concave down

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval [0, 4). The function f is twice differentiable except at $x=2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that their derivatives of f do not exist and $x=2$.

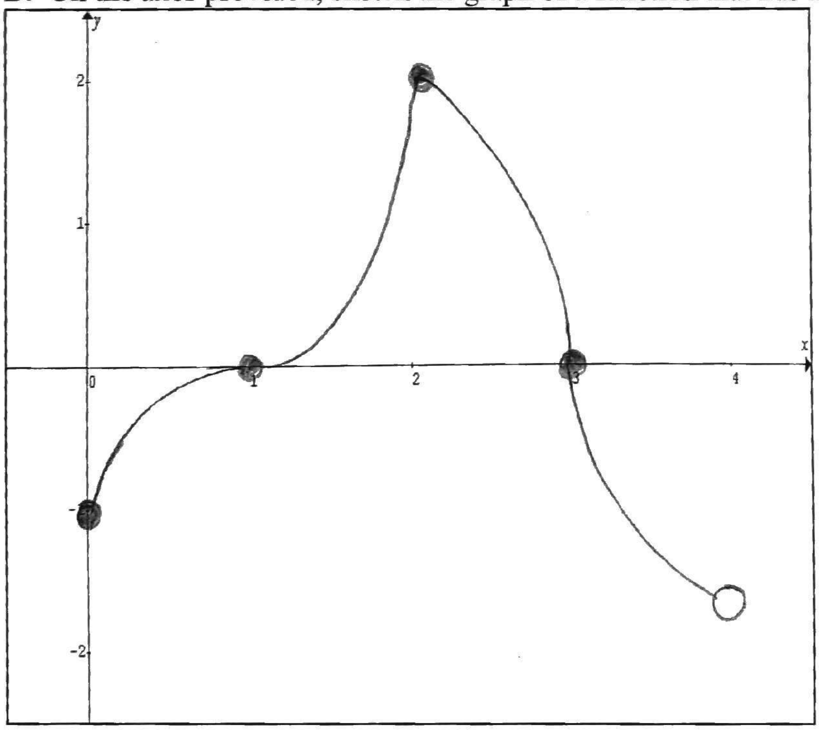
$f(2)$ exists

A. For $0<x<4$, find all values of x at which f has a local extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

$x=2 \rightarrow$ local max at $x=2$ b/c $f'(x)$ goes from positive to negative at $x=2$.

also state: $f'(x)$ DNE at $x=2$

B. On the axes provided, sketch the graph of a function that has all the characteristics of f.



1 Continued...

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval [0, 4). The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that their derivatives of f do not exist and $x = 2$.

C. Let g be the function defined by $g(x) = \int_1^x f(t)dt$ on the open interval (0,4). For $0 < x < 4$, find all values at which g has a relative extremum. Determine whether g has a relative maximum or relative minimum at each of these values. Justify your answer.

$g'(x) = f(x)$ [FTC I]

$g'(x) = f(x) = 0$ at $x = 1$ and $x = 3$

$x = 1$: g has a relative min at $x = 1$ b/c $g'(x)$ goes from negative to positive at $x = 1$

$x = 3$: g has a relative max at $x = 3$ b/c $g'(x)$ goes from positive to negative at $x = 3$

D. For the function g defined in part C, find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

$g''(x) = f'(x)$

$g''(x) = f'(x) = 0$ at $x = 1$

$g''(x) = f'(x) \Rightarrow$ DNE at $x = 2$

only at 2

g has a point of inflection at $x = 2$ b/c $g''(x) = f'(x)$ changes sign at $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

2. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at select values of x. The function h is given by $h(x) = f(g(x)) - 6$.

A. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

B. Explain why there must be a value of c for $1 < c < 3$ such that $h'(c) = -5$.

C. Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value of $w'(3)$.

D. If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

A) $r=1$: $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

Since h is continuous and goes from 3 to -7 it must pass through -5 for some value of r. (Intermediate Value Thm)

B] Ave rate Δ in h over $(1,3)$ $= \dfrac{h(3) - h(1)}{3-1} = \dfrac{-7-3}{2} = \dfrac{-10}{2} = -5$

Since h is cont. and differentiable, there must exist a value c on $(1,3)$ such that $h'(c) = -5$ (Mean Value Thm)

C] $w'(x) = f(g(x)) \cdot g'(x)$

$w'(3) = f(g(3)) \cdot g'(3)$

$= f(4) \cdot (2)$

$= -1 \cdot 2$

$= -2$

D] $\frac{d}{dx}[g^{-1}(2)] = \dfrac{1}{g'(1)} = \dfrac{1}{5}$

$g(1) = 2$

$g^{-1}(2) = 1$

$\boxed{y - 1 = \frac{1}{5}(x-2)}$

3. Let f be the function given by $f(x)=\dfrac{\ln x}{x}$ for all $x>0$. The derivative of f is given by

$f'(x)=\dfrac{1-\ln x}{x^2}$.

A. Write an equation for the line tangent to the graph of f at $x=e^2$

B. Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, relative maximum, or neither for the function f. Justify your answer.

C. The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

A] $f'(e^2)=\dfrac{1-\ln(e^2)}{e^4}=\dfrac{-1}{e^4}=-e^{-4}$ $\qquad f(e^2)=\dfrac{\ln(e^2)}{e^2}=\dfrac{2}{e^2}$

$$y-\frac{2}{e^2}=\frac{-1}{e^4}(x-e^2)$$

B] $f'(x)=\dfrac{1-\ln x}{x^2}=0 \Rightarrow 1-\ln x=0 \Rightarrow x=e$

(*) $\quad + + + + \quad e \quad - - - - \quad f'(x)$

f has a local max at $x=e$ b/c $f'(x)$ goes from positive to negative at $x=e$.

C]

$$f''(x)=\frac{x^2\left(-\frac{1}{x}\right)-(1-\ln x)(2x)}{(x^2)^2}=\frac{-x-2x+2x\ln x}{x^4}$$

$$=\frac{-3x+2x\ln x}{x^4}=0$$

$3\cancel{x}=2\cancel{x}\ln x \qquad x\neq 0$

$\ln x=\dfrac{3}{2} \qquad x=e^{3/2}$

we do not have to check if $f''(x)$ changes sign b/c the question assured us there was one point of inflection. This is the only candidate.