AB.Q403.REVIEW ASSESSMENT
(Part E)

PARTICLE MOVEMENT (VELOCITY)

(28 points)

CALCULATORS PERMITTED

[**Decimal Answers – Round to Three Decimal Places**]

NAME:

DATE: Key

BLOCK:

I (*print name*)________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. A particle moves up and down along the y-axis. This particle has position $y=-4$ at $t=0$ seconds. The velocity of the particle is defined as $v(t)=t\cdot\tan^{-1}(t)-t^2+2t+2$ m/s.

A. What is the acceleration of the particle at $t=2$?

$$v'(2) = -0.493 \ m/s^2$$

B. Find all values of t on $0<t<5$ where in instantaneous velocity is equal to the average velocity over the interval $0<t<5$. Justify.

$$\text{Ave velocity}: \frac{\int_0^5 v(t)\,dt}{5-0} = \frac{1}{5}\int_0^5 v(t)\,dt = 1.738 \ m/s$$

$$v(t) = 1.738 \ \text{at} \ t = 3.359 \ s$$

C. For what value(s) of t on $0<t<5$ is the speed of the particle equal to 2? Justify.

$$v(t) = 2 \ \text{at} \ t = 3.274$$

$$v(t) = -2 \ \text{at} \ t = 4.276$$

D. What is the position of the particle at $t=3$? Is the particle moving up or down the y-axis at this time? Justify.

$$\int_0^3 v(t)\,dt = y(3) - y(0) \rightarrow y(3) = -4 + \int_0^3 v(t)\,dt = 6.745$$

$v(3) = 2.747 > 0$ $\therefore$ The particle is moving up the y-axis

E. What is the position of the particle at $t=6$? Is there some time on $0<t<6$ for which the particle will be at the same position in started? Justify your answer?

$$\int_3^6 v(t)\,dt = y(6) - y(3) \rightarrow y(6) = 6.745 + \int_3^6 v(t)\,dt = -4.996$$

OPTION 1: $v(t)$ is continuous and on $[3,6]$ $v(t)$ goes from 6.745 to -4.996 and so, by the Intermediate Value Thm $v(t)$ must pass through -4 at some point on this interval.

OPTION 2: Simply find at $t = 5.925$ $v(t) = -4$

2. A particle moves left and right long the x-axis. The particle has a velocity of 5 when $t = 0$. The acceleration of the particle is defined as $a(t) = -3t - 10\sin(t^2 - t)$.

What is the velocity of the particle at time $t = 2$? Is the particle speeding up or slowing down at $t = 2$? Justify your answer.

What is the velocity of the particle at time $t = -2$? Is the particle speeding up or slowing down at $t = -2$? Justify your answer.

$\int_0^2 a(t)\,dt = V(2) - V(0)$: $\quad V(2) = 5 + \int_0^2 a(t)\,dt = -5.488$

$a(2) = -15.093$

The particle is speeding up at $t=2$ because the velocity and acceleration share the same sign at $t=2$

$\int_{-2}^0 a(t)\,dt = V(0) - V(-2)$: $\quad V(-2) = 5 - \int_{-2}^0 a(t)\,dt = 2.501$

$a(-2) = 8.794$

The particle is speeding up at $t=-2$ b/c the velocity and acceleration share the same sign at $t=-2$.

3. At time $t = 0$, particle **Q** starts at $x = 3.5$ and moves horizontally along the line $y = 2$ with a constant velocity of $q(t) = 0.4912$ m/s.

At time $t = 0$, particle **P** starts at $x = 2.5$ and moves horizontally along the x-axis with a velocity of $p(t) = 2 + t - t\ln(t+1)$ m/s.

The distance between particle **P** and particle **Q** at $t = 0$ is $\sqrt{5}$ m.

Keep $q(t) = 0.4912$ *but round decimal answers to three decimal places.*

Area or |absolute Value|

A. Find the total distance traveled by each particle on $0 \le t \le 6$.

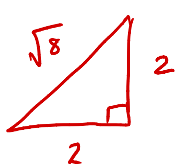
$$P: \int_0^6 |p(t)|\,dt = \int_0^{3.681} p(t)\,dt - \int_{3.681}^{6} p(t)\,dt = 10.050$$

$$Q: \int_0^6 |q(t)|\,dt = \int_0^6 0.4912\,dt = 2.947$$

B. What is the distance between particle **P** and particle **Q** at time $t = 6$ seconds.

$$P(6): \int_0^6 p(t)\,dt = P(6) - P(0) \ : \ P(6) = 2.5 + \int_0^6 p(t)\,dt = 4.447 \leftarrow \text{Position of } P$$

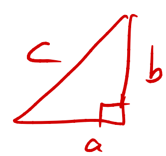
$$Q(6): \int_0^6 q(t)\,dt = Q(6) - Q(0) \ : \ Q(6) = 3.5 + \int_0^6 q(t)\,dt = 6.447 \leftarrow \text{Position of } Q$$



$D = \sqrt{8}$ m

C. What is the rate of change in the distance between the two particles at time $t = 6$ seconds?

Rell $\quad a^2 + b^2 = c^2$



$$\square\ 2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$\square\ 2(2)(4.1662) + 2(2)(0) = 2\sqrt{8}\frac{dc}{dt}$$

$p'(6) = -3.675$

$q'(6) = 0.4912$

$$\frac{da}{dt} = |p'(6) - q'(6)| = 4.1662$$

$$\boxed{\frac{dc}{dt} = 2.946}$$

4. A particle starts at $x = 6$ and moves along the x-axis. Its velocity, in feet per second, was recorded at several times as shown in the chart below. Note: v is a differentiable function of t.

t (seconds)	$v(t)$ (feet per second)
0	10
3	4
5	-1
10	-2
12	9

A. Approximate $\int_0^{12} |v(t)| dt$ using a trapezoidal sum with 4 intervals. Using correct units, explain the meaning of the integral.

$$\frac{10+4}{2} \cdot 3 + \frac{4+1}{2} \cdot 2 + \frac{1+2}{2} \cdot 5 + \frac{2+9}{2} \cdot 2$$

This is the total distance in feet traveled by the particle over $t=0$ to $t=12$ seconds

average on a small neighborhood

B. Approximate the acceleration of the particle at $t = 3.7$ seconds. Indicate the units.

$$v'(3.7) \approx \frac{-1-(-4)}{5-3} = \frac{-5}{2} \text{ ft/sec}^2$$

C. Find the average acceleration of the particle over the interval $0 \le t \le 12$.

$$\frac{v(12)-v(0)}{12-0} = \frac{9-10}{12} = \frac{-1}{12} \text{ ft/sec}^2$$

D. Based on the values in the table, what is the smallest number of instances at which the acceleration of the particle could equal zero on the open interval $0 < t < 12$. Justify.

on $[3,5]$ average rate $\Delta = -5/2$

on $[5,10]$ average rate $\Delta = -1/5$

on $[10,12]$ average rate $\Delta = \frac{11}{2}$

$v(t)$ is differentiable so $v(t)$ is continuous and so by the M.V.T $v'(t)$ must equal $-\frac{1}{5}$ and $11/2$ at some point on their respective intervals

Now, since $a(t)$ goes from negative to positive it must pass through zero at least on time by the Intermediate Value Theorem.