



AB.Q403.REVIEW ASSESSMENTS

(PART D)

DERIVATIVES AND APPLICATIONS

(28 points)

NO CALCULATORS

NAME:

DATE:

BLOCK:

Key

I (print name) _____ certify that I wrote and fully understand **all** marks made in this assessment. I did not write anything that I do not understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. Let $f(x) = \begin{cases} 5 - (1-x)^{5/2}; & x < 1 \\ 6 - (2-x)^2; & x \geq 1 \end{cases}$

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute maximum and the absolute minimum value of f on the closed interval $-1 \leq x \leq 4$. Box your answer. Show the analysis that leads to your conclusion.

a) i) $f(1) = 6 - (2-1)^2 = 5$

ii) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 6 - (2-x)^2 = 5$

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 5 - (1-x)^{5/2} = 5$

$\therefore \lim_{x \rightarrow 1} f(x) = 5$ * It is not sufficient to just say the limit exists

iii) $\lim_{x \rightarrow 1} f(x) = f(1)$ $\therefore f$ is continuous at $x = 1$

b) $f'(x) = \begin{cases} \frac{5}{2}(1-x)^{3/2} & \text{for } x < 1 \\ 2(2-x) & \text{for } x > 1 \end{cases}$ * $f'(x)$ DNE at $x = 1$
but the function $f(x)$ is continuous at $x = 1$

interior critical x-values $\left[\begin{array}{l} f'(x) = 0 \text{ at } x = 2 \\ f'(x) \text{ DNE at } x = 1 \end{array} \right.$

endpoint $\left[x = -1, x = 4 \right.$

$f(-1) = 5 - 2^{5/2}$

$f(1) = 5$

$f(2) = 6$

$f(4) = 2$

absolute max = 6
absolute min = $5 - 2^{5/2}$

or $5 - \sqrt{32}$
or $5 - 4\sqrt{2}$

multiplication ... not addition

2. Use Lagrange (prime) notation to express the following derivatives

A. $[f(g(x)+2)]' = f'(g(x)+2) \cdot g'(x)$

B. $[\ln(f(x))+g(2x)]' = \frac{1}{f(x)} \cdot f'(x) + g'(2x) \cdot 2$

$[f^2(x)] = [f(x)]^2$

C. $[f^2(x) \cdot h(g(x))]' = f^2(x) \cdot h'(g(x)) \cdot g'(x) + h(g(x)) \cdot 2 f(x) \cdot f'(x)$
Product Rule

3. Let $f(x) = -x^5 - x^3 - x - 5$ and $g(x)$ be the inverse function of f . Find $g'(-2)$.

$$g'(2) = \frac{1}{f'(-1)} = \frac{1}{-5(-1)^4 - 3(-1)^2 - 1} = -\frac{1}{9}$$

$$-2 = -a^5 - a^3 - a - 5$$

$$3 = -a^5 - a^3 - a$$

guess and check

$$a = -1$$

4. Find the derivative of $\sin^{-1}(2x^2)$

Book of memories

$$\frac{1}{\sqrt{1-(2x^2)^2}} \cdot 4x$$

Product Rule
(x · y)

5. Consider the curve C: $\sin(xy) + 2y = x + y^2 + \frac{4-\pi}{4}$

A. Find $\frac{dy}{dx}$.

B. Find the equation of the line tangent to the curve at the point $\left(\frac{\pi}{4}, 2\right)$

$$\cos(xy) \cdot \left[x \frac{dy}{dx} + y(1) \right] + 2 \frac{dy}{dx} = 1 + 2y \frac{dy}{dx}$$

$$\underline{x \cos(xy) \frac{dy}{dx}} + \underline{y \cos(xy)} + \underline{2 \frac{dy}{dx}} = \underline{1} + \underline{2y \frac{dy}{dx}}$$

$$\frac{dy}{dx} (x \cos(xy) + 2 - 2y) = \underline{1 - y \cos(xy)}$$

$$\frac{dy}{dx} = \frac{1 - y \cos(xy)}{x \cos(xy) + 2 - 2y}$$

$$\left. \frac{dy}{dx} \right|_{\left(\frac{\pi}{4}, 2\right)} = \frac{1}{2-4} = -\frac{1}{2}$$

$$\boxed{y - 2 = -\frac{1}{2} \left(x - \frac{\pi}{4}\right)}$$

6. Consider the curve C: $\sqrt{3}y + 2\sin y = 1 + x^3$ for $0 \leq y \leq \pi$.
Find a point on the graph of C where the tangent line to C is vertical.

$$\sqrt{3} \frac{dy}{dx} + 2 \cos(y) \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{\sqrt{3} + 2\cos y} \quad \text{vertical means } \frac{dy}{dx} \text{ DNE}$$

Set denominator = zero

$$\sqrt{3} + 2\cos y = 0 \quad \cos y = -\frac{\sqrt{3}}{2} \quad y = \frac{5\pi}{6}$$

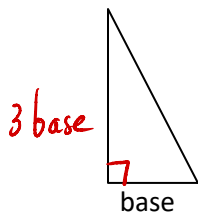
Now find x ...

$$\sqrt{3} \frac{5\pi}{6} + 2 \sin\left(\frac{5\pi}{6}\right) = 1 + x^3$$

$$x^3 = \frac{5\sqrt{3}\pi}{6} \quad x = \sqrt[3]{\frac{5\sqrt{3}\pi}{6}}$$

$$\left(\sqrt[3]{\frac{5\sqrt{3}\pi}{6}}, \frac{5\pi}{6} \right)$$

7. A chemical leak in the corner of a science laboratory room makes the shape of a right triangle on the floor. The triangle grows in such a way that the height is always three times the base. The length of the base is growing at the rate $7/10$ feet per second at the very instant the base is $5/3$ feet in length.



Given $\frac{db}{dt} = \frac{7}{10} \text{ ft/sec}$ when $b = \frac{5}{3}$

A. Find the rate at which the area of the triangle is growing at the instant the base is $5/3$ feet in length. Include the units of measure.

Find $\frac{dA}{dt}$

Rel $A = \frac{1}{2}(b)(3b) = \frac{3b^2}{2}$

$\frac{dA}{dt} = 3b \frac{db}{dt}$



$\left. \frac{dA}{dt} \right|_{b=5/3} = 3\left(\frac{5}{3}\right)\left(\frac{7}{10}\right) = \frac{7}{2} \text{ ft}^2/\text{sec}$



B. Find the rate at which the hypotenuse of the right triangle is growing at the instant the base is $5/3$ feet in length. Include the units of measure.

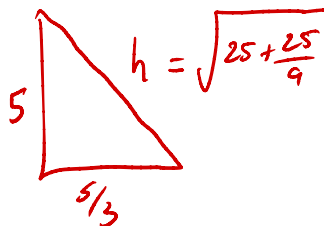
Find $\frac{dh}{dt}$

Rel $b^2 + (3b)^2 = h^2$

$b^2 + 9b^2 = h^2$

$10b^2 = h^2$

$20b \frac{db}{dt} = 2h \frac{dh}{dt}$



$2\left(\frac{5}{3}\right)\left(\frac{7}{10}\right) = 2\left(\sqrt{25 + \frac{25}{9}}\right)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{20\left(\frac{5}{3}\right)\left(\frac{7}{10}\right)}{2\sqrt{25 + \frac{25}{9}}} = \frac{\frac{35}{3}}{\frac{5\sqrt{10}}{3}} = \frac{7}{\sqrt{10}} \text{ ft/sec}$
or $\frac{7\sqrt{10}}{10}$

C. The rate of change of the total temperature of the triangle is modeled by the function

$T'(t) = t(t^2 + 1)^4$ units per second. Based on this model, what was the temperature increase from time one to two seconds?

$$\int_1^2 T'(t) dt = \int_1^2 t(t^2 + 1)^4 dt = \frac{1}{2} \int_2^5 u^4 du$$

$$u = t^2 + 1$$

$$du = 2t dt$$

$$dt = \frac{du}{2t}$$

$$= \frac{1}{2} \left[\frac{u^5}{5} \right]_2^5$$

$$= \frac{1}{10} (t^2 + 1)^5 \Big|_1^2$$

$$= \frac{1}{10} \left[(5)^5 - (2)^5 \right]$$

$$= \frac{3093}{10} \text{ units}$$