AB.Q403.REVIEW ASSESSMENT (PART C)

DIFFERENTIAL EQUATIONS

(28 points)

NO CALCULATOR

NAME: Key

DATE:

BLOCK:

I (*print name*)___________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. Evaluate A. $\int e^{5x}\,dx$ B. $\int e^{\frac{x}{3}}\,dx$ C. $\int 2\sin(7x)\,dx$ D. $\int 2\cos\left(\frac{x}{9}\right)dx$

A] $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$

why? $u = 5x$

$du = 5\,dx$

$dx = \frac{du}{5}$

$\int e^{5x}\,dx = \frac{1}{5}\int e^{u}\,du = \frac{1}{5}e^{5x} + C$

B] $\int e^{x/3}\,dx = 3e^{x/3} + C$

C] $\int 2\sin(7x)\,dx = -\frac{2}{7}\cos(7x) + C$

D] $\int 2\cos\left(\frac{x}{9}\right)dx = 18\sin\left(\frac{x}{9}\right) + C$

2.

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = 6(14 - y)$ with $f(0) = 2$.

A. Find $\frac{d^2y}{dx^2}$ in terms of y.

B. Use part A to find $f''(0)$.

C. Solve the differential equation by separating the variables.

$* \ \frac{d}{dx}(-6y) = -6 \cdot \frac{d}{dx}(y) = -6\frac{dy}{dx}$

Key point

I distributed

A] $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(6(14-y)\right) = \frac{d}{dx}(84-6y) = -6\frac{dy}{dx} = \boxed{-6(84-6y)}$

B] $f''(0) = f''(0,2) = -6\left(84-6(2)\right) = \boxed{-432}$

C] $\int \frac{dy}{14-y} = \int 6\,dx$

$-\ln|14-y| = 6x + C$

$\ln|14-y| = -6x + C$

$|14-y| = Ce^{-6x}$

$14 - y = C^* e^{-6x}$

$y = 14 - C^* e^{-6x}$

$2 = 14 - C^* \quad \therefore C^* = 12$

$\boxed{y = 14 - 12e^{-6x}}$

Very important negative

It is a major mistake to miss this negative

3.
Let $y = H(t)$ be the particular solution to the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(10 - H) \text{ with } H(0) = 2.$$

A. Use a linearization centered at $t = 0$ to approximate the value of $H(0.75)$

B. Solve the differential equation by separating the variables.

A]

$$L(t) = H(0) + H'(0,2)(t - 0)$$

$$\boxed{L(t) = 2 + 4(t)}$$ tangent line

$$H(0.75) \approx L(0.75) = 2 + 4(0.75) = \boxed{5}$$

B]

$$\int \frac{dH}{10-H} = \int \frac{1}{2}\, dt$$

$$-\ln|10-H| = \frac{1}{2}t + c$$

again

$$\ln|10-H| = -\frac{1}{2}t + c$$

$$|10-H| = Ce^{-\frac{1}{2}t}$$

$$10 - H = c^* e^{-\frac{1}{2}t}$$

$$H = 10 - c^* e^{-1/2\, t}$$

$$2 = 10 - c^* \quad \therefore c^* = 8$$

$$\boxed{H = 10 - 8e^{-t/2}}$$

4.

Let $y = f(t)$ be the particular solution to the differential equation

$\frac{dy}{dt} = \frac{1}{3}y(12-y)$ with $f(0) = 5$. also $\left(4y - \frac{y^2}{3}\right)$ "distribution"

A. Find $f'(0)$

B. Find $f''(0)$.

C. Find $\lim_{t\to 0^+} \frac{f(t)-5}{\sin(t/3)}$

A] $f'(0) = f'(0,5) = \frac{1}{3}(5)(7) = \frac{35}{3}$

B] $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(4y - y^2/3\right) = 4\frac{dy}{dx} - \frac{2}{3}y\frac{dy}{dx}$

C] $f''(0) = f''\left(x=0, y=5, \frac{dy}{dx} = \frac{35}{3}\right) = \left(4\,\frac{35}{3} - \frac{2}{3}\cdot 5\cdot\frac{35}{3}\right) = \frac{35}{3}\left(4 - \frac{2}{3}(5)\right) = \frac{70}{9}$

D] $\lim_{t\to 0^+} \frac{f(t)-5}{\sin(t/3)} \left\{\frac{0}{0}\right\} = \lim_{t\to 0^+} \frac{f'(t)}{\frac{1}{3}\cos(t/3)} = \frac{35/3}{\frac{1}{3}(1)} = \boxed{35}$

L'HOPITAL'S RULE

$\lim_{t\to 0^+} f(t) - 5$ AND $\lim_{t\to 0^+} \sin(t/3) = 0$ * Never say $= \frac{0}{0}$ AP will take off points

$$y - y_1 = m(x - x_1)$$
$$y = y_1 + m(x - x_1)$$

5.

Let $\frac{dy}{dx} = \frac{(1-y)}{x^2}$ where $y = f(x)$ is the particular solution to the differential equation with the condition $f(2) = 0$.

A. Use the line tangent to f at $x = 2$ to approximate $f(1)$

B. Solve the differential equation by separating the variables.

A]
$$L(x) = f(2) + f'(2,0)(x-2)$$
$$L(x) = 0 + \frac{1}{4}(x-2)$$
$$f(1) \approx L(1) = \frac{1}{4}(-1) = \boxed{\frac{-1}{4}}$$

B]
$$\int \frac{dy}{1-y} = \int \frac{dx}{x^2} \xrightarrow{} \int x^{-2}\,dx$$
$$-\ln|1-y| = \frac{x^{-1}}{-1} + c$$
$$\ln|1-y| = x^{-1} + c$$
$$|1-y| = c\,e^{x^{-1}}$$
$$1 - y = c^* e^{x^{-1}}$$
$$y = 1 - c^* e^{x^{-1}}$$

$$0 = 1 - c^* e^{1/2}$$
$$c^* = \frac{1}{e^{1/2}} = e^{-1/2}$$
$$y = 1 - e^{-1/2} e^{x^{-1}}$$

or

$$y = 1 - \frac{1}{e^{1/2}} e^{x^{-1}}$$

$$y = 1 - \frac{1}{e^{1/2}} \cdot e^{\frac{1}{x}}$$

$$y = 1 - e^{\left(\frac{1}{x} - \frac{1}{2}\right)}$$

Variety of ways to write the answer!

6.
Let $y = f(x)$ be the particular solution to the differential equation
$\frac{dy}{dx} = \cos x + 2y^2$ with $f(0) = -3$.

A. Find $f^{/}(0)$
B. Find $f^{//}(0)$.
C. $\lim_{x \to 0} \frac{f(x) - 19x + 3}{5x^2}$

A] $f'(0) = f'(0,-3) = \cos(0) + 2(9) = \boxed{19}$

B] $f''(x,y) = -\sin x + 4y \underbrace{\frac{dy}{dx}}_{\text{sub}} = -\sin x + 4y \underbrace{(\cos x + 2y^2)}_{\text{sub}}$

$f''(0) = f''(0,-3) = -\sin(0) + 4(-3)(1+18) = \boxed{-228}$

C] $\lim_{x \to 0} \frac{f(x) - 19x + 3}{5x^2} \left\{\frac{0}{0}\right\} = \lim_{x \to 0} \frac{f'(x) - 19}{10x} \left\{\frac{0}{0}\right\} = \lim_{x \to 0} \frac{f''(x)}{10} = \frac{-228}{10} = \boxed{-22.8}$

L'HOPITAL'S RULE

L'HOPITAL'S RULE

$\lim_{x \to 0} f(x) - 19x + 3 = 0$ and $\lim_{x \to 0} 5x^2 = 0$

$\lim_{x \to 0} f'(x) - 19 = 0$ and $\lim_{x \to 0} 10x = 0$