Advanced Calculus

AB.Q403.REVIEW ASSESSMENT (PART B)

THE FUNDMENTAL THEOREM OF CALCULUS

(42 points)

NO CALCULATOR

ANSWERS SHEET

NAME: Key

DATE:

BLOCK:

I (*print name*)________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

QUESTION 1

A. Find $g(2)$, $g'(2)$, and $g''(2)$.

$g(2) = \int_{-1}^{2} f(x)dx = \frac{1+3}{2}(2) + \frac{1}{2}(3) = \boxed{4 + \frac{3}{2}} = \frac{11}{2}$

$g'(2) = f(2) = \boxed{0}$ ← point on graph of f

$g''(2) = f'(2) = \boxed{-3}$ ← slope of graph of f

B. On what interval between $-5 < x < 5$ is g increasing? Justify your answer.

$(-5, 2]$ or $(-5, 2)$ because $g'(x) = f(x) > 0$ on $(-5, 2)$

C. For what values of x is $g(x)$ concave downward? Justify your answer.

$(-3, -1) \cup (1, 3)$ because $g''(x) = f'(x) < 0$ on this interval

D. Write the equation of any horizontal tangents to $g(x)$ between $-5 < x < 5$.

$g'(x) = f(x) = 0$ at $x = 2$ $\quad g(2) = \frac{11}{2}$ $\quad y - \frac{11}{2} = 0(x-2)$

or

$y = \frac{11}{2}$

E. Find the absolute minimum value of $g(x)$ on the interval $-5 \le x \le 5$. Justify your answer.

$\underbrace{g'(x) = 0 \text{ at } x = 2}_{\text{you must communicate this}}$ ← local max here

$g(5) = \int_{-1}^{5} f(x)dx = 4 + \frac{3}{2} - \frac{3}{2} - 6 = -2$

$g(-5) = \int_{-1}^{-5} f(x)dx = -\int_{-5}^{-1} f(x)dx = -\left[4 + \frac{2+1}{2} \cdot 2\right] = \boxed{-7}$ ← absolute min

F. Find the average rate of change of $g'(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of c, for $-5 < c < 5$, such that $g''(c)$ is equal to this average rate of change $g'(x)$? Why or why not? If so, find a value of c that satisfies the conclusion of the mean value theorem.

ave rate Δ of $g'(x)$ = ave. rate Δ of $f(x)$ $= \dfrac{f(5)-f(-5)}{5-(-5)}$

$= \dfrac{-3-2}{10} = \dfrac{-5}{10} = -\dfrac{1}{2}$

The mean value theorem does not guarantee b/c $f(x)$ is not differentiable on $(-5,5)$. Namely, $f(x)$ has a corner at $x = -3, -1, 1, 3$.

In order to guarantee $f(x)$ must be both continuous on $[-5,5]$ and differentiable on $(-5,5)$

G. Find the average value of $f(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of z, for $-5 < z < 5$, such that $f(z)$ is equal to this average value of $f(x)$? Why or why not? If so, find a value of z that satisfies the conclusion of the mean value theorem.

Ave. value of $f(x)$ $= \dfrac{\int_{-5}^{5} f(x)\,dx}{5-(-5)} = \dfrac{1}{10}\int_{-5}^{5} f(x)\,dx$

$= \dfrac{1}{10}\left[4+3+4+\dfrac{3}{2}-\dfrac{3}{2}-6\right] = \dfrac{5}{10} = \dfrac{1}{2}$

The mean value theorem (for integrals) does guarantee b/c $f(x)$ is continuous on $[-5,5]$ $\therefore$ (This is the only requirement for M.V.T for Integrals)

H. Let $h(x) = 2x^2 - \int_{-1}^{x} f(t)\,dt$. Find $h'(3)$.

$h'(x) = 4x - f(x)$

$h'(3) = 4(3) - f(3) = 12 - (-3) = \boxed{15}$

I. Let $p(x) = \int_{-1}^{4x^3+2} f(t)\,dt$. Find $p'(-1)$.

$p'(x) = f(4x^3+2)(12x^2)$

$p'(-1) = f(-2)\cdot 12(-1)^2 = \dfrac{3}{2}(12) = \boxed{18}$

QUESTION 2

A. Estimate $g(8)$ using a right Riemann sum with three rectangles.

$$g(8)=\int_0^8 f(x)dx \approx 14(2)+12(5)+10(1) = 98$$

B. Estimate $g(-5)$ using a trapezoidal approximation with three trapezoids.

$$g(-5)=\int_0^{-5} f(x)dx = -\int_{-5}^{0} f(x)dx = -\left[\frac{10+(-3)}{2}(3)+\frac{(-3)+4}{2}(1)+\frac{4+8}{2}(1)\right]$$

$$= -\left[\frac{-13}{2}\cdot 3 + \frac{1}{2} + 6\right]$$

C. Find $g'(-1)$

$$g'(-1)=f(-1) = 4$$

D. Estimate $g''(2.6)$.

$$g''(2.6) = f'(2.6) \approx \frac{f(7)-f(2)}{7-2} = \frac{12-14}{5} = \frac{-2}{5}$$

E. If $m(x)=\int_0^{1-\ln x} f(t)dt$, find $m'(e)$.

$$m'(x) = f(1-\ln x)\cdot \frac{-1}{x} \qquad m'(e) = f(1-\cancel{\ln e}^{\,1})\cdot\frac{-1}{e} = f(0)\,\frac{-1}{e} = \frac{-8}{e}$$

F. If $b(x)=g\left(\frac{x}{4}\right)$, find $b'(8)$.

$$b(x)=g\left(\frac{x}{4}\right)=\int_0^{x/4} f(t)dt \quad b'(x)=f\left(\frac{x}{4}\right)\cdot\frac{1}{4} \quad b'(8)=f(2)\cdot\frac{1}{4} = \frac{14}{4} = \frac{7}{2}$$

G. What is the minimum number of times that $f'(x)=0$ on $-5 \le x \le 8$. Make use of the appropriate theorems to justify your answer.

minimum of one time.

$f(x)$ is continuous on $[-5,8]$ ✓

$f'(x)$ is differentiable on $(-5,8)$ ✓

on $[0,2]$ ave $f'(x) = \frac{6}{2}=3$ $f'(x)=3$ at some point on $(0,2)$ by M.V.T

on $[2,7]$ ave $f'(x) = \frac{-2}{5}$ $f'(x) = \frac{-2}{5}$ at some point on $(2,7)$ by M.V.T

$f'(x)$ is continuous on $[0,7]$

$f'(x)$ goes from positive to negative at least on time on $[0,7]$

$\therefore$ by I.V.T $f'(x)=0$ at least on time.

QUESTION 3

A. The graph of $y = f(x)$ is decreasing for what x-values?

$[-3,-1] \cup (1,5]$ or $(-3,-1) \cup (1,5)$

B. Compute $f(5)$ and $f(-3)$

$$\int_0^5 f'(x)\,dx = f(5) - f(0) \longrightarrow (1-2-1) = f(5) - 12 \quad \therefore f(5) = 12 + 1 - 2 - 1 = 10$$

$$\int_{-3}^0 f'(x)\,dx = f(0) - f(-3) \longrightarrow f(-3) = \boxed{12 - \left(-\frac{1}{3} + 2 - 3\right) - \frac{1}{2}} = \frac{77}{6}$$

$$f(-3) = f(0) - \int_{-3}^{0} f'(x)\,dx$$

$$\int_{-3}^{-1} f'(x)\,dx = \int_{-3}^{-1} (x^2 + 4x + 3)\,dx = \left[\frac{x^3}{3} + 2x^2 + 3x\right]_{-3}^{-1} = \left(-\frac{1}{3} + 2 - 3\right) - (-9 + 18 - 9)$$

$$\int_{-1}^{0} f'(x)\,dx = \frac{1}{2} \leftarrow \text{area of triangle}$$

C. Find the absolute maximum and absolute minimum values of $f(x)$ on $-3 \le x \le 5$. Justify your answers.

Interior critical x values : $f'(x)=0$ at $x=-1$
$f'(x)$ DNE at $x=1$

you must state this ... it justifies why they are critical x-values

endpoint x values $x=-3,\ x=5$

$f(-3)=\frac{77}{6}$

$f(-1)=f(0)-\int_{-1}^{0} f'(x)\,dx = 12-\frac{1}{2}=\frac{23}{2}$

$f(1)=f(0)+\int_{0}^{1} f'(x)\,dx = 12+1=13$

$f(5)=10$

$\boxed{\begin{array}{l}\text{abs max} = 13\\ \text{abs min} = 10\end{array}}$

D. Find $\lim_{x\to 0}\frac{f(x)-12}{1-\ln(x+e)}$ Show work.

$$\lim_{x\to 0}\frac{f(x)-12}{1-\ln(x+e)}\ \left\{\frac{0}{0}\right\} = \lim_{x\to 0}\frac{f'(x)}{-\frac{1}{x+e}} = \frac{f'(0)}{-\frac{1}{e}} = \frac{1}{-\frac{1}{e}} = \boxed{-e}$$

L'HOPITAL'S RULE

$\lim_{x\to 0} f(x)-12=0$

also

$\lim_{x\to 0} 1-\ln(x+e)=0$

E. Find $\lim_{x\to 1^+}\frac{f'(x)+x}{x^2-1}$ Show work.

$$\lim_{x\to 1^+}\frac{f'(x)+x}{x^2-1}\ \left\{\frac{0}{0}\right\} = \lim_{x\to 1^+}\frac{f''(x)+1}{2x} = \frac{0+1}{2} = \boxed{\frac{1}{2}}$$

L'HOPITAL'S Rule

$\lim_{x\to 1^+} f'(x)+x=0$

also

$\lim_{x\to 1^+} x^2-1=0$