AB.Q403.REVIEW ASSESSMENT (Part A)

RATE IN / RATE OUT

(X points)

SOLUTIONS

CALCULATORS PERMITTED

[**Decimal Answers – Round to Three Decimal Places**]

NAME:

DATE:

BLOCK:

I (*print name*)_________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. A 20-gallon kiddie pool has 15 gallons of water at 8:00 am ($t = 0$ hours). A hole in the pool makes the water drain at the rate of $f(t) = 13 \cdot \ln(t+2)$ gallons per hour.
At 8:00 am, a dad puts a hose into the pool which adds water at the rate

$$r(t) = \begin{cases} e^{-0.05t}; & 0 \le t \le 0.5 \\ 25; & t > 0.5 \end{cases}$$ gallons per hour.

A. Find $f(0.25)$ and $f'(0.25)$. Include the units of measure for each.

$f(0.25) = 10.542$ gallons/hr $\quad f'(0.25) = 5.778$ gallons/hr^2

B. How much water drained from the pool between 8:00am and 8:30am?

$\int_0^{0.5} f(t)\,dt = 5.258$

C. How much water is in the pool at 8:30am?

$A(0.5) = 15 + \int_0^{0.5} r(t)\,dt - \int_0^{0.5} f(t)\,dt = 10.236$ gallons

D. Write an equation for the amount of water in the pool at any time t between 8:00am and 8:30am.

$A(t) = 15 + \int_0^t [r(u) - f(u)]\,du$ must use a dummy variable

E. In the amount of water in the kiddie pool increasing or decreasing at 8:15am ($t = 0.25$)? Justify.

$f(0.25) = 10.542 > r(0.25) = 0.988 \leftarrow$ I show this to justify

decreasing b/c rate out at $t = 0.25$ is greater than rate in

F. In the amount of water in the kiddie pool increasing or decreasing at 9:15am ($t = 1.25$)? Justify.

$f(1.25) = 15.323 < r(1.25) = 25 \leftarrow$ I show this to justify

increasing b/c rate the rate in at $t = 1.25$ is greater than rate out

G. How much water was added to the pool between 8:00am and 9:00am?

$\int_0^{0.5} e^{-0.05t}\,dt + \int_{0.5}^{1} 25 = 0.494 + 12.5$ or 12.994 gallons

H. How much water is in the pool at 9:00am?

$A(1) = 10.236 - \int_{0.5}^{1} f(t)\,dt + \int_{0.5}^{1} 25\,dt$

$10.236 - 6.566 + 12.5$ or 16.170 gallons

I. Set up, but do not solve, an equation used to find the answer to the following question:
If the water hose is not removed from the pool, then when will it begin to overflow?

$16.170 + \int_1^t 25\,du - \int_1^t f(u)\,du = 20$

rate in

rate out

2. An oil tank contains 200 cubic meters of oil at $t=0$. Oil is pumped into the tank at a rate $E(t)=2+\sin(t)\cdot\tan^{-1}(t)$ m^3 per hour and leaves at the rate $L(t)=\frac{t}{4}+0.5$ m^3 per hour.

A. What is the average rate of change in the amount of oil in the tank for $0\le t\le 8$ hours? Include units of measure.

Ave value of rate = $\dfrac{\int_0^8 [E(t)-L(t)]\,dt}{8-0} = \dfrac{4.80292}{8} = 0.600 \ m^3/hr$

B. What is the absolute maximum and absolute minimum amount of oil in the tank on the time interval $0\le t\le 8$ hours? Show the analysis that leads to your conclusion.

$A'(t) = \underbrace{E(t)-L(t)=0}$ at $t=3.617$, $t=6.344$

must show this to justify interior critical

$A(0) = 200$

$A(3.617) = 200 + \int_0^{3.617}[E(t)-L(t)]\,dt = 205.521$

$A(6.344) = 200 + \int_0^{6.344}[E(t)-L(t)]\,dt = 203.646$

$A(8) = 200 + \int_0^{8}[E(t)-L(t)]\,dt = 204.803$

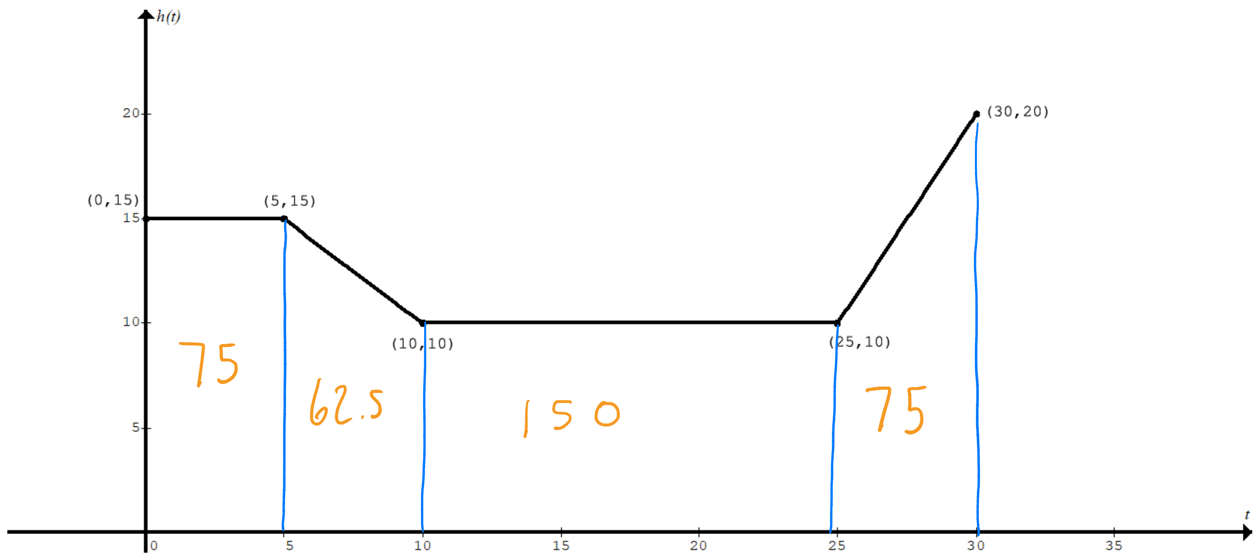
$$\boxed{\begin{array}{l}\text{ABS MIN} = 200 \ m^3 \\ \text{ABS MAX} = 205.521 \ m^3\end{array}}$$

3. A tank initially holds 400 cubic feet of water.

Water enters a tank at the rate $g(t)$ cubic feet per hour for $0 \le t \le 30$ hours.
Select values of $g(t)$ are shown in the table below.

t	0	5	10	15	20	25	30
$g(t)$	21	15	16	14	25	35	12

Water drains from the tank at the rate $h(t)$ cubic feet per hour for $0 \le t \le 30$ hours.
The graph of $y = h(t)$ is made up of line segments and is shown in the graph below.



A. Approximate the total amount of water that enters the tank between 0 and 30 hours using a **trapezoidal** sum with six equal intervals. Simplify.

$$\int_0^{20} g(t)\,dt \approx \frac{5}{2}\left(21 + 2(15) + 2(16) + 2(14) + 2(25) + 2(35) + 12\right)$$

$$= 607.5 \text{ ft}^3$$

B. How much water left the tank over the first 30 hours?

$$\int_0^{30} h(t)\,dt = 15(5) + \frac{15+10}{2}(5) + 15(10) + \frac{10+20}{2}(5) = 362.5 \text{ ft}^3$$

C. Using the estimate in part A, find the amount of water in the tank at time $t = 30$ hours.

$$A(30) = 400 + \int_0^{30} g(t)\,dt - \int_0^{30} h(t)\,dt$$

$$= 400 + 607.5 - 362.5$$

$$= 645 \text{ ft}^3$$

D. Is water volume increasing or decreasing at time $t = 10$ hours? Justify.

$g(10) = 16 > h(10) = 10$ ← show this to justify

increasing b/c rate in at $t=10$ is greater than rate out

E. Estimate the value of $g'(27)$ and find the value of $h'(27)$. Include units of measure for each.

$g'(27) \approx \frac{g(30) - g(25)}{30 - 25} = \frac{12 - 35}{5}$ ft^3/hr^2

-4.6

g is ft^3/hr

g/time ft^3/hr^2

$h'(27) = 2$ ft^3/hr^2

F. Using the correct units, find and interpret the value of $\frac{1}{30}\int_0^{30} h(t)\,dt$ in the context of the problem.

Average rate of water flowing out of the pipe (ft^3/hr) over the first 30 hrs. (or from $t=0$ to $t = 30$ hrs)

$\frac{362.5}{30} = 12.083$ ft^3/hr

G. Using the correct units, find and interpret the value of $\frac{1}{30}\int_0^{30} g'(t)\,dt$ in the context of the problem.

Average rate of change in the rate of water flowing into the pipe (ft^3/hr^2) over the first 30 hrs.

$\frac{12-21}{30} = -0.3$ ft^3/hr^2