AB.Q403.REVIEW ASSESSMENT (PART F)

Derivative Tests and Miscellaneous Concepts

(20 points)

NO CALCULATORS

NAME:

DATE:

BLOCK:

I (*print name*)________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

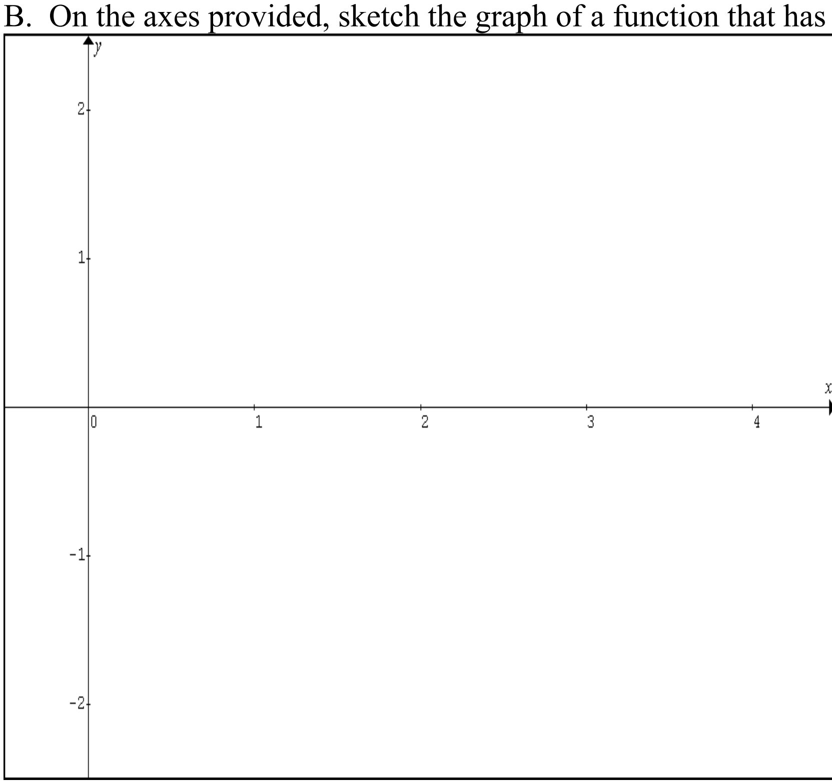
1.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let *f* be a function that is continuous on the interval [0, 4). The function *f* is twice differentiable except at $x = 2$. The function *f* and its derivatives have the properties indicated in the table above, where DNE indicates that their derivatives of *f* do not exist and $x = 2$.

A. For $0 < x < 4$, find all values of *x* at which *f* has a local extremum. Determine whether *f* has a relative maximum or a relative minimum at each of these values. Justify your answer.

B. On the axes provided, sketch the graph of a function that has all the characteristics of *f*.



1 Continued…

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

Let *f* be a function that is continuous on the interval [0, 4). The function *f* is twice differentiable except at $x = 2$. The function *f* and its derivatives have the properties indicated in the table above, where DNE indicates that their derivatives of *f* do not exist and $x = 2$.

C. Let *g* be the function defined by $g(x) = \int_1^x f(t)dt$ on the open interval (0,4). For $0 < x < 4$, find all values at which *g* has a relative extremum. Determine whether *g* has a relative maximum or relative minimum at each of these values. Justify your answer.

D. For the function *g* defined in part C, find all values of *x*, for $0 < x < 4$, at which the graph of *g* has a point of inflection. Justify your answer.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

2. The functions *f* and *g* are differentiable for all real numbers, and *g* is strictly increasing. The table above gives values of the functions and their first derivatives at select values of *x*. The function *h* is given by $h(x) = f(g(x)) - 6$.

A. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

B. Explain why there must be a value of c for $1 < c < 3$ such that $h'(c) = -5$.

C. Let *w* be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value of $w'(3)$.

D. If g^{-1} is the inverse function of *g*, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

3. Let *f* be the function given by $f(x)=\frac{\ln x}{x}$ for all $x>0$. The derivative of *f* is given by $f'(x)=\frac{1-\ln x}{x^2}$.

A. Write an equation for the line tangent to the graph of *f* at $x=e^2$

B. Find the *x*-coordinate of the critical point of *f*. Determine whether this point is a relative minimum, relative maximum, or neither for the function *f*. Justify your answer.

C. The graph of the function *f* has exactly one point of inflection. Find the *x*-coordinate of this point.