AP CALCULUS AB: Q302: DIFFERENTIAL EQUATIONS and SLOPE FIELDS

Solve the differential equation.

1. $\frac{dy}{dx} = 3\sqrt{xy}$ where $y = 256$ when $x = 4$

2. $y' = \frac{xy}{8}$

3. $f'(x, y) = e^{x-y}$

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A slope field is a lattice of line segments on the Cartesian plane that indicate the slope of a function or other curve at the designated points if the curve were to go through the point.

Ex: 1 $\frac{dy}{dx} = 3x^2$.

Sketch the **slope field** on the given points and find the general solution to the differential equation.

Ex: 2. $\frac{dy}{dx} = \frac{2x}{3y}$.

Sketch the **slope field** on the given points and find the general solution to the differential equation

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Derive a formula for a **linear approximation** (a tangent to a function or curve used to estimate the value of the function or curve)

Derive a formula for **Euler's approximation** (a series of line segments that use the slope of the line or function used to estimate the value of the function or curve)

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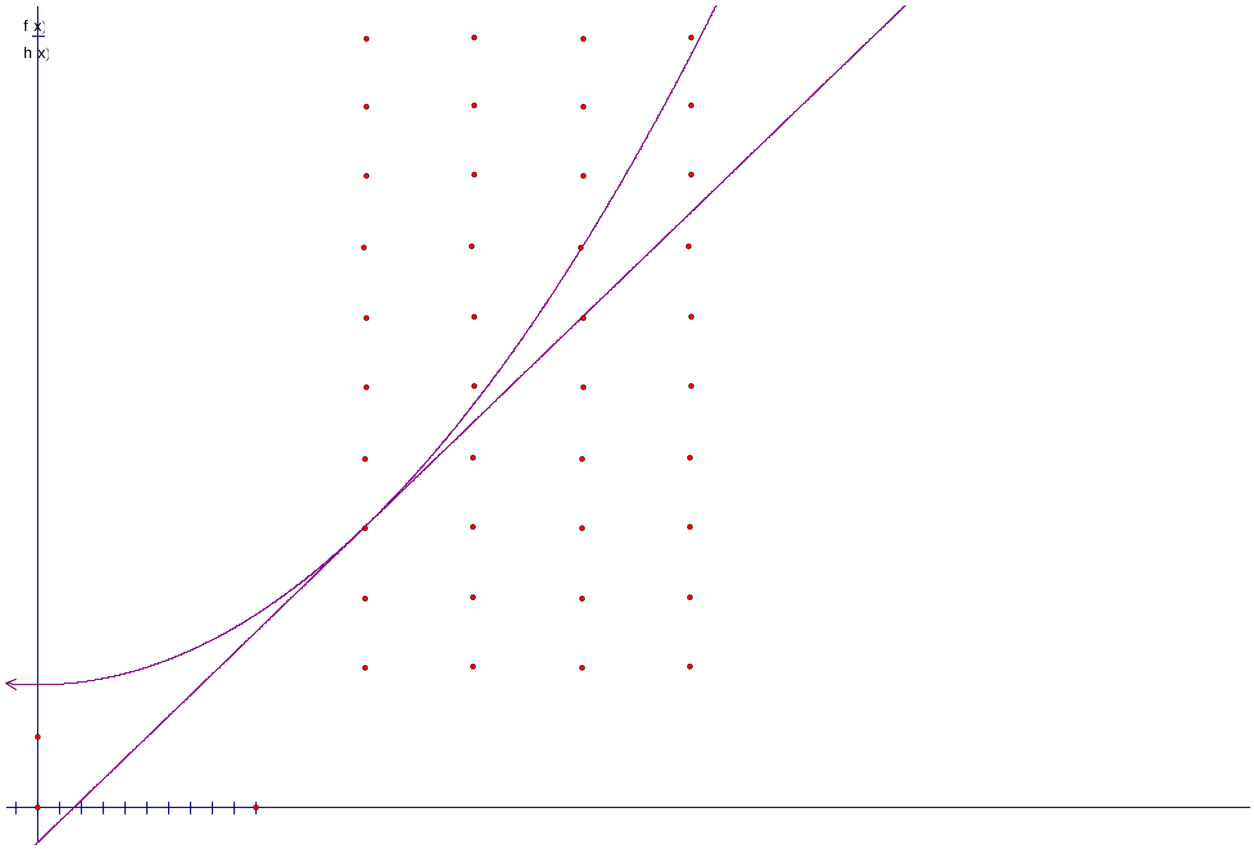

1A. Suppose $\frac{dy}{dx} = 2x$ with $y(1.5) = 4$. Write an equation of a line tangent to the graph of y at $x = 1.5$ and use it to approximate $y(3)$.

1B. Use Euler's Method to approximate $y(3)$ starting with $y(1.5)=4$ and using $\Delta x = 0.5$.

1C. Find the exact solution to the differential equation $\frac{dy}{dx} = 2x$ with $y(1.5) = 4$.

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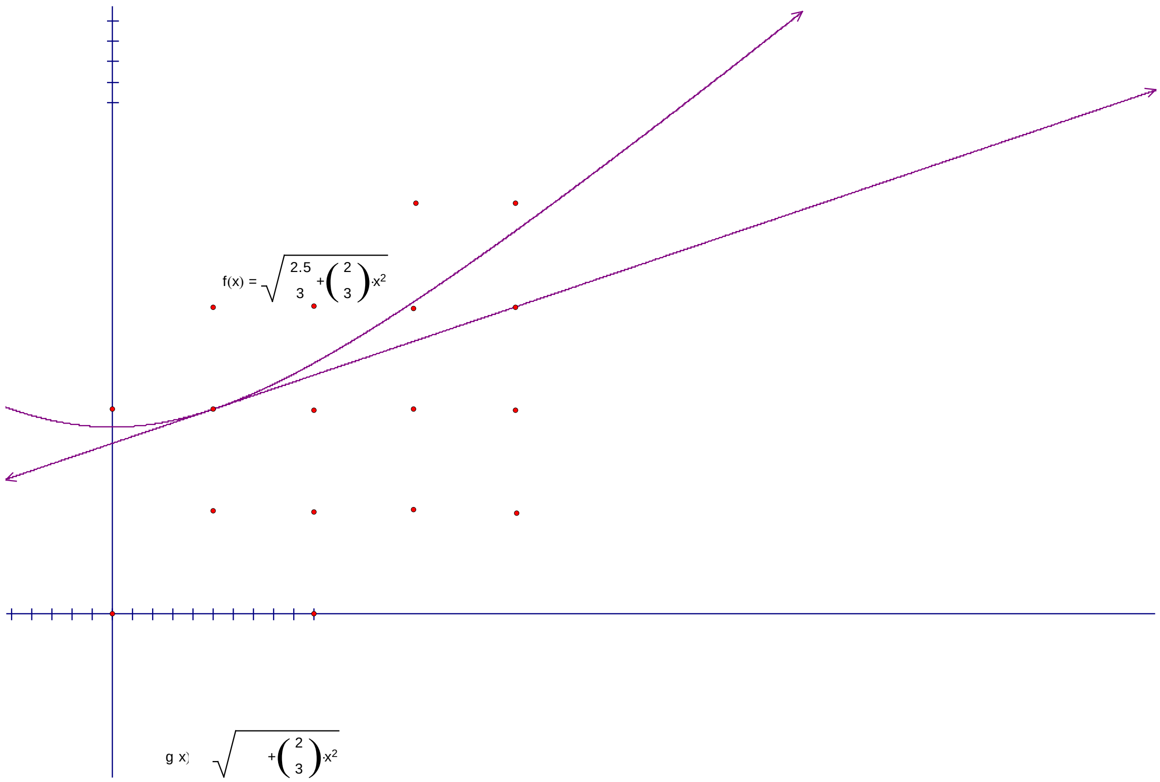

1A. Suppose $\frac{dy}{dx} = \frac{2x}{3y}$ where the point (0.5, 1) lies on the curve. Write an equation of a line tangent to the graph of y at $x = 0.5$ and use it to approximate the positive value of y when x = 1.5.

1B. Use Euler's Method to approximate the positive value of y when x = 1.5 starting with the point (0.5, 1) and using $\Delta x = 0.5$.

1C. Find the exact solution (the positive value of y when x = 1.5) to the differential equation $\frac{dy}{dx} = \frac{2x}{3y}$ where point (0.5, 1) lies on the curve.

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Practice 1: Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where x = 1.
(b) Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2)
(c) Find f(x) by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with initial condition $f(1) = 4$.
(d) Use your solution from part (c) to find the exact solution to f(1.2).

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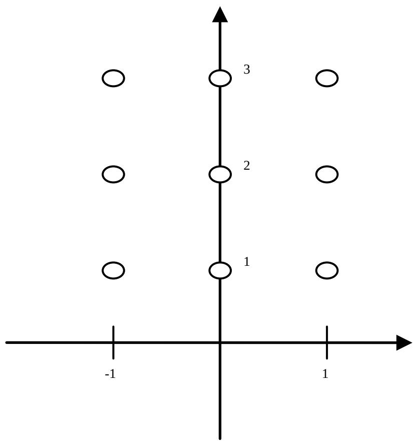
Practice 2: Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Let $y = f(x)$ be a particular solution to the given differential equation with initial condition $f(0) = 3$. Use a linear approximation centered at x = 0 to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(0) = 3$. Use the solution to find $f(0.2)$.

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Practice 3: Let f be a function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \dfrac{1+e^x}{x^2}$.

(a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$ Use f'' to explain why this approximation is less than $f(3.1)$.

(b) Use Euler's method, starting at $x = 3$ with step size 0.05 , to approximate $f(3.1)$.

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Chapter 6.4, 6.5

Warm Up:

Solve the differential equation $\frac{dy}{dt} = ky$ subject to the initial condition $y(0) = y_0$. k is a constant.

Comment:

Solve the differential equation $\frac{dy}{dt} = k(L - y)$ subject to the initial condition $y = y_0$ when $t = 0$. L and k are constants.

Comment:

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Exponential Growth and Decay
Differential equation $\frac{dy}{dt} = ky$ subject to $y = y_0$ at $t = 0$ is $y = y_0 e^{kt}$

Example 1: Solve the differential equation $\frac{dy}{dt} = 0.04y$ given that $y_0 = 15$. Find $y(6)$.

Example 2: (Biological Growth) History indicates that the population y of the world (during the last 200 years) has been growing at a rate proportional to the population y with the growth constant $k = 0.0198$. The population of the world was about 4 billion on January 1, 1975. When will the world's population reach 10 billion?

Example 3: (Biological Growth) The cells in a certain bacterial culture divide (double) on average every 2.5 hours. If there were 500 cells initially, how many cells would we expect to find after 12 hours?

Example 4: (Radioactive Decay) Radium has a half-life of 1690 years. Determine the decay constant k for the radium and then calculate how much of 10 grams of radium will be left after 2400 years (Assume that radium decays at a rate proportional to the amount of radium present with some decay constant k). When will the amount remaining be 1 gram?

Example 5: (Carbon Dating) Human hair from a grave in Africa proved to have only 74% of the carbon 14 found in living tissue. Given that carbon 14 has a half-life of 5570 years, determine when the body died. (Assume the exponential decay model)

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Simple Bounded Growth (Newton's Law of Cooling)

Definition: The solution to the differential equation $\frac{dy}{dt} = k(L - y)$ subject to $y = y_0$ at $t = 0$ is $y = L - (L - y_0)e^{-kt}$.

Example 6: Solve the differential equation $\frac{dy}{dt} = 0.4(10 - y)$ subject to $y_0 = 3$.

Example 7: The news that the mayor of a certain city had been killed was announced at noon, and in 3 hours it was thought that 75% of the people in the city had heard it. How long will it take for 99% of the people to hear it?

Example 8: The classic model for fish growth assumes that the rate of change in fish length is proportional to the difference between theoretical maximum length and actual length. A certain variety of fish is hatched at length 0.5 inches and never grows beyond 12 inches. If a typical such fish has a length of 6 inches in 20 weeks, how long will it be after 50 weeks?

Example 9: A hard-boiled egg at 98°C is put in a pan under running 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C?

Example 10: A cup of water with a temperature of 95°C is placed in a room with constant temperature of 21°C. (a) Assuming that Newton's Law of cooling applies, set up and solve an initial-value problem whose solution is the temperature t minutes after it is placed in the room. (b) How many minutes will it take for the water to reach a temperature of 51°C if it cools to 85°C in one minute?

Example 11: Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$ where the constant of proportionality is k. (a) if $P(0)=500$, find $P(t)$ in terms of t and k. (b) If $P(2) = 700$, find k. (c) find $\lim_{t\to\infty} P(t)$.